Federal Reserve Credibility and the Term Structure of Interest Rates*

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Abstract

A setting with optimal monetary policy is susceptible to the well-known time-inconsistency problem, where the credibility of the central bank plays a crucial role. First, in a simple model of optimal monetary policy we show how different assumptions about the level of credibility can have important implications for the term structure of interest rates. Next, using a regime-switching approach, we estimate a medium-scale DSGE model with optimal monetary policy where bond prices are consistent with the agents’ inter-temporal marginal rate of substitution. Federal Reserve behavior is modeled using the flexible loose commitment setting, which nests the commonly used discretion and full commitment assumptions. We find that neither full commitment nor discretion can do a satisfactory job of explaining term structure dynamics. We investigate the historical effects of re-optimization shocks and find that they can act as both expansionary and contractionary shocks in affecting yields, with a sizable impact on the level, slope and curvature of the yield curve.

Keywords: Term Structure, Commitment, Regime-Switching Bayesian Estimation, Optimal Monetary Policy, DSGE models

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1 Introduction

How does monetary policy impact the term structure of interest rates? The answer to this question is of interest to central bankers who want to understand how their actions affect long-term interest rates and consequently the economy. This topic is also relevant for bond market participants so that they can make informed investment decisions. Thus it is no surprise that there is a growing literature that tries to analyze this issue.\(^1\) As is common in the monetary policy literature most of these analyses use a simple Taylor rule to model monetary policy. But as macroeconomic models become more sophisticated, increasing attention is being paid to the modeling of optimal monetary policy. However, an optimal policy framework with forward looking agents gives rise to the time-inconsistency issue, which is well known since the work of Kydland and Prescott (1977) and Barro and Gordon (1983). The policy maker can reap the benefits of shaping agents’ expectations by announcing a plan and credibly committing to it. But this policy is not time-consistent as the policy maker has an ex-post incentive to deviate from the promised plan. The optimal monetary policy literature has dealt with this issue by assuming either that the central bank has access to a commitment technology (full commitment case) or that they re-optimize every period (discretion case). Yet neither of the two dichotomous cases of discretion or full commitment seems reasonable in practice.\(^2\) Moreover, recent theoretical and descriptive evidence suggests that assumptions about central bank credibility may have a key effect on the term structure.\(^3\) In this paper we use the general framework of loose commitment (this nests both the full commitment and discretion cases) and explore both theoretical and empirical implications for the term structure of interest rates.

We begin by considering a simple theoretical model to shed light on the effects of optimal monetary policy on the term structure. This analysis generalizes the work of Palomino (2012) where only discretion and commitment are considered. We use the framework of loose commitment, following the work of Roberds (1987), Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010). This is a flexible setting in which the central bank has the ability to commit to its future plans, but it may occasionally give in to the temptation to re-optimize plans. These re-optimization episodes are modeled using a regime-switching

\(^1\)See recent papers by Campbell et al. (2014), Ang et al. (2011) and Bikbov and Chernov (2013) and references therein.

\(^2\)In an empirical study with a medium scale DSGE model, Debortoli and Lakdawala (forthcoming) show that both full commitment and discretion are rejected by the data.

\(^3\)See the analyses of Palomino (2012) and Campolmi et al. (2012) for a detailed discussion.
process where both the policy maker and the agents are aware of the possibility and take it into account when forming expectations. We embed optimal monetary policy within the loose commitment framework in a simple New-Keynesian model where a cost-push shock drives the dynamics. The degree of credibility has a key effect on the covariance between the agents’ stochastic discount factor and bond returns. This in turn determines whether long-term bonds are viewed by investors as acting as a hedge or increasing their risk. The typical assumption of full commitment or discretion can have stark implications for the yield curve. In contrast, the loose commitment setting provides a more flexible framework where different values for the degree of credibility can generate a wide variety of properties for the yield curve. The loose commitment framework also affects the dynamic behavior of the yield curve through the effects of re-optimization shocks. The response of the economy and bond prices to re-optimization shocks is history dependent and this setting can help generate rich and complicated dynamics for the entire yield curve.

Having highlighted the main mechanisms in the simple model, we then estimate a fully specified medium scale dynamic stochastic general equilibrium (DSGE) model using U.S. data on both macroeconomic variables and bond yields. The analysis is conducted in the model based on the work of Smets and Wouters (2007). We depart from that model in two important ways. First, monetary policy is conducted by a central bank operating under loose commitment, rather than being described by a simple interest rate rule. Second, we augment the model with yield data and derive bond prices that are consistent with the stochastic discount factor of the agents. The degree of credibility affects the agents’ expectations for both macro variables and bond prices, and is a parameter of the model that is estimated. The presence of re-optimization shocks generates regime-switching dynamics in the state variables. We derive bond prices in this framework using a log-linear approximation as in Bansal and Zhou (2002) and Ang et al. (2008) among others. Additionally, for estimation purposes this requires the use of regime-switching techniques. We use a Bayesian Markov Chain Monte Carlo procedure following Debortoli and Lakdawala (forthcoming).

The degree of credibility of the Federal Reserve is estimated to be 0.86, similar to the results of Debortoli and Lakdawala (forthcoming). The use of quarterly data implies that re-optimizations are expected to occur once every 7 quarters. An advantage of the estimation framework is that it allows for the identification of historical episodes when the Federal Reserve likely abandoned its commitments, as measured by the (smoothed) probability of re-optimization. We find that policy re-optimizations likely occurred throughout
the sample with the exception of three long periods (mid 1980s to early 1990s, late 1990s and early 2000s till 2007). Using impulse responses we emphasize the history dependence in the effects of re-optimization shocks. The effect of a re-optimization that is preceded by a markup shock makes yields lower relative to the case where no re-optimization occurs, while a re-optimization preceded by a technology shock has the opposite effect. An analysis of the historical effects of re-optimizations in U.S. data reveals that they acted as expansionary monetary shocks before 1990s and as contractionary monetary shocks after that. The contemporaneous effect of re-optimizations is larger for medium maturities than at either the short or long end of the yield curve. To understand the effects on the entire yield curve we construct simple measures representing three factors that are commonly used in the literature: level, slope and curvature. Comparing the model implied effects of re-optimization shocks to the data, we notice that re-optimizations have a sizeable contemporaneous effect on the curvature of the yield curve, while the effects on the level and the slope peak at around the two year mark.

The model does a good job of fitting the term structure data, especially at the shorter end of the yield curve. However, the model fails to generate enough volatility for long-term yields and we discuss potential ways of addressing this issue. With the rich DSGE model, we can perform a structural decomposition of the shocks contributing to the yield curve. We find that demand, markup and monetary policy shocks are the main drivers of bond yields, while technology shocks have a limited influence. Finally, we conduct a counterfactual analysis to explore how yields would have behaved under different credibility scenarios. We find that neither full commitment nor discretion can satisfactorily characterize the yield dynamics captured by the loose commitment setting. Our results suggest that under discretion, average bond yields would have been much lower than the data. On the other hand, a full commitment framework would generate a much flatter slope of the yield curve relative to that data. We conclude that the flexibility of the loose commitment framework helps significantly in explaining term structure data from the perspective of a structural macro model.

There is a growing empirical literature that evaluates the time-inconsistency issue in optimal monetary policy. See Debortoli and Lakdawala (forthcoming), Chen et al. (2013), Matthes (2015), and Coroneo et al. (2013) among others. But these papers only consider macro models without any implications for the term structure of interest rates. For the empirical section of our paper we build on the loose commitment framework of Debortoli
and Lakdawala (forthcoming) and embed the yield curve. In this respect, our work is related to the macro-finance literature that tries to combine structural macro models with the term structure of interest rates; see Gürkaynak and Wright (2012) and Duffee (2015) for excellent surveys of recent studies. Early work like Hordahl et al (2006) and Rudebusch and Wu (2008) combined simple New-Keynesian models with an ad hoc stochastic discount factor to price long-term bonds. A more recent empirical study using the same approach is Bikbov and Chernov (2013) where they consider regime switching in the monetary policy rule and volatility of shocks. In contrast, here we derive the stochastic discount factor and the implied bond price dynamics that are consistent with the inter-temporal marginal rate of substitution of the representative household. Starting with Bekaert et al. (2010), there are several empirical studies that use this approach. Two notable recent papers are Amisano and Tristani (2009) and Chib et al. (2012) which study the term structure of interest rates using a regime-switching DSGE model. We extend their analysis in two important ways. First, instead of a simple interest rate rule, we consider optimal monetary policy and use identified re-optimization episodes to provide a more structural interpretation of shifts in monetary policy behavior. Second, we use a richer DSGE model that can better account for the dynamic properties of a variety of macroeconomic variables. The paper most similar to ours is Palomino (2012), which examines bond yield dynamics implied by optimal monetary policy in a general equilibrium model. The main advantage of our approach is that the loose commitment framework allows for the full continuum of values for policy credibility, while Palomino (2012) only considers two extreme cases: discretion and full commitment. Additionally, while Palomino (2012) relies on a calibration exercise to examine the empirical implications of the theoretical model, we use regime-switching Bayesian methods to estimate our DSGE model using data on macro variables and bond yields.

Recent work studying yield curve properties in DSGE models has been able to generate a time-varying term premium. In our current setting, the re-optimization shock is i.i.d. This assumption combined with the linearized DSGE model results in a constant term premium even in the presence of regime-switching dynamics in the state variables. The i.i.d. assumption is necessary to keep the model tractable for estimation. While we recognize this limitation of our framework, the focus of the paper is to empirically evaluate the effects of

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Federal Reserve credibility on the term structure, using the flexible loose commitment setting. To that end, ours is the first paper to estimate a DSGE model with optimal monetary policy and the term structure.

The remainder of the paper is divided in two main parts. In the next section we describe the loose commitment framework and use a simple model to explain the basic conceptual issues involved in optimal monetary policy setting in this framework and its implications for the yield curve. In Section 3 we start with a brief overview of the DSGE model and the estimation algorithm. Next we present the results from the estimation and the key term structure results. Finally, we offer some concluding remarks in section 4.

2 Loose Commitment and the Yield Curve

In this section, we first explain the intuition behind loose commitment in a simple macro model. Then we add bond yields and show how term structure properties are related to the degree of credibility and how bond yields respond to re-optimization shocks.

2.1 The Loose Commitment Setting

The working assumption is that the central bank has access to a commitment technology, but it occasionally succumbs to the temptation to revise its plans, termed as policy re-optimizations. This is similar to the assumption in Schaumburg and Tambalotti (2007) and Debertoli and Nunes (2010). Private agents are aware of the possibility of policy re-optimizations and take it into account when forming expectations. More formally, at any point in time, monetary policy can switch between two alternative scenarios, captured by the unobserved state variable $s_t \in \{0, 1\}$. If $s_t = 1$, previous commitments are honored. Instead, if $s_t = 0$, the central bank makes a new (state-contingent) plan over the infinite future, disregarding all the commitments made in the past. The variable $s_t$ evolves according to a two-state stochastic process

\[
 s_t = \begin{cases} 
 1 \text{ with prob. } \gamma \\
 0 \text{ with prob. } 1 - \gamma 
\end{cases}
\]
In the limiting case where the probability $\gamma = 1$, the central bank always honors its promises and this formulation coincides with the canonical full commitment case. Instead if $\gamma = 0$, the central bank always re-optimizes, as in the approach commonly referred to as discretion. The main advantage of this setup is that $\gamma$ can take on any value in $[0, 1]$ and can be estimated from the data. Note that the switching is i.i.d. in nature. This means that the probability of a re-optimization occurring next period is the same, regardless of a re-optimization having occurred in the current period or not. Debortoli and Lakdawala (forthcoming) provide a discussion and some suggestive evidence in support of this assumption. From the perspective of asset pricing, this assumption has important implications that are discussed in section 2.

In the case of the Federal Reserve these re-optimizations could represent a change in the composition of the Federal Open Market Committee (the Fed’s main policy making arm) due to appointment of a new chairman or a change in the voting members. Additionally pressure from the political system or the financial markets may cause a re-optimization. The results in Debortoli and Lakdawala (forthcoming) suggest that the Federal Reserve does not have full credibility but that it can be viewed as being close with $\gamma$ estimated to be around 0.8. This is consistent with the empirical results presented in section 2.3.

### 2.2 Loose Commitment in a Simple Model

The main conceptual issues behind the loose commitment framework are illustrated using a simple model similar to Clarida et al. (1999) and also used by Palomino (2012). Consider a quadratic loss function for the central bank, where the aim is to minimize deviations of inflation ($\pi_t$) and output gap ($y_t$) from their target levels. Without loss of generality, the targets for both are assumed to be equal to zero.

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \kappa y_t^2 + \theta \pi_t^2 \right]$$

(1)

This loss function is minimized subject to constraints that govern the dynamics of inflation and output gap.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t$$

(2)

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1})$$

(3)
The first equation is commonly referred to as the New-Keynesian Phillips Curve and can be derived from optimal firm pricing behavior. $u_t$ is a cost-push shock (also known as markup shock) that is modeled as an i.i.d. process without loss of generality. The second equation is called the dynamic IS curve and can be derived from the household optimization problem, where $i_t$ is the nominal short interest rate. This equation can be appended with a shock that could be interpreted as a demand shock. However, this demand shock does not create a tradeoff for the policymaker and would contribute nothing to the analysis at hand. In this setup, it is the cost-push shock that creates a tradeoff between inflation and output gap stabilization.\(^5\) In other words, without the presence of $u_t$ optimal policy can be achieved by setting both $\pi_t$ and $y_t$ to zero for each time period. While in the presence of this shock, the central bank is not always able to simultaneously set both $\pi_t$ and $y_t$ to zero. It must choose the relevant tradeoff which depends on the state of the economy and the central bank preference parameters. The first order condition for optimal policy under discretion can be represented by

$$\pi_t^d = \frac{1}{\theta} y_t^d \tag{4}$$

This equation depicts the classic principle of “leaning against the wind” and adjusting inflation in the opposite direction to the deviation of output gap from its target.\(^6\) The relationship under full commitment is given by

$$\pi_t^c = \frac{1}{\theta} [y_t^c - y_{t-1}^c] \tag{5}$$

This equation is similar to the discretion case but now inflation responds to the change in the output gap rather than the level of the output gap. When formulating optimal policy under commitment, the central bank takes into account the effects of their policy on agents’ expectations. This effect is ignored in the discretion case. To get a better understanding, figure 1 shows the impulse responses to an i.i.d. cost-push shock using the following calibration: $\kappa = .25$, $\beta = 0.99$, $\theta = 1$ and $\sigma = 2$. Under both discretionary and commitment regimes, inflation rises on impact and the output gap falls. Under the discretionary case inflation is high just

\(^5\) As discussed in Clarida et al. (1999), a central bank without commitment is not able to smooth out the effects of economic fluctuations, thus giving rise to the so-called “stabilization bias”, which is the source of time-inconsistency in our setting. Note that this time-inconsistency issue occurs even in the absence of an “inflation bias” that arises when the central bank wishes to push output above its natural level.

\(^6\) The $d$ superscript denotes the dynamics of the variables when policy is conducted under discretion. Similarly we will use the superscripts $c$ and $lc$ for full commitment and loose commitment respectively.
on impact (period 1) and falls back to zero (the target value) from the next period. In the commitment case, the central bank promises to lower inflation in the future and thus the rise in inflation on impact is not as high relative to the discretionary case. Under discretion the short rate has to be raised enough contemporaneously to completely absorb the effects of the cost-push shock, while under commitment the short rate does not have to be raised as much on impact and then it is gradually moved back to zero.

The key point is that agents should trust the central bank to be credible and follow through with the promise of low inflation from period 2 onwards, after the central bank has reaped the reward of lower relative inflation in period 1. This is the crux of the time-inconsistency issue and creates the incentive for the policy maker to re-optimize. If the central bank is not perfectly credible then we are in the loose commitment setting and agents assign a positive probability of the central bank reneging on its promises in any period in the future. Under loose-commitment, the dynamics are affected by a re-optimization shock \( s_t \) in addition to the cost-push shock. The relations implied by the first order conditions are now given by

\[
\pi^l_t = \begin{cases} 
-\frac{1}{\bar{\sigma}} y^l_t, & \text{if } s_t = 0 \\
-\frac{1}{\bar{\sigma}} (y^l_t - y^l_{t-1}), & \text{if } s_t = 1 
\end{cases}
\] (6)

Figure 2 shows the effect of the i.i.d cost-push shock under the loose commitment setting with probability of commitment \( \gamma = 0.5 \). This means that agents expect that a re-optimization will occur in any period with probability 0.5. The thin blue line shows the response of the variables in the case that no re-optimization shock occurs. While the blue line with the crosses shows the behavior when a re-optimization shock occurs in period 3. The thin blue line shows that the central bank promised to keep inflation low for a few periods after the cost-push shock but with a re-optimization this promise is not kept and inflation is set to 0, which minimizes the central bank’s current period loss. The inflation response under loose commitment lies in between the discretion and full commitment cases for the first few periods. However the dynamic behavior of the variables under loose commitment do not have to lie in between discretion and full commitment as can be seen more clearly from the response of the output gap and the short rate. The crucial determinants of the effects of re-optimization shocks are the timing of the re-optimization shock and the history of all other shocks preceding the re-optimization shock.\(^7\)

Before we derive bond prices in the simple model, we setup the general formulation

\(^7\)An implication of this is that if a re-optimization shock occurs in the steady-state, it will have no effect.
of optimal policy in the loose commitment framework. Gathering all the state variables \([y_t, \pi_t, i_t, u_t]\) in \(x_t\) and the exogenous shock \([u_t]\) in \(v_t\), the system of equations can be written as

\[
A_{-1}x_{t-1} + A_0x_t + A_1E_t x_{t+1} + Bv_t = 0 \tag{7}
\]

We can write the optimization problem for the central bank in the following format.

\[
x'_{t-1}Vx_{t-1} + d = \min_{\{x_t\}_{t=0}^\infty} E_{-1} \sum_{t=0}^\infty (\beta \gamma)^t [x'_t W x_t + \beta(1 - \gamma)(x'_t V x_t + d)] \tag{8}
\]

\[
s.t. A_{-1}x_{t-1} + A_0x_t + \gamma A_1E_t x_{t+1} + (1 - \gamma)A_1E_t x_{t+1}^{reop} + Bv_t = 0 \ \forall t \tag{9}
\]

The terms \(x'_{t-1}Vx_{t-1} + d\) summarize the value function at time \(t\). Since the problem is linear quadratic, the value function is given by a quadratic term in the state variables \(x_{t-1}\), and a constant term \(d\) reflecting the stochastic nature of the problem. The objective function is given by an infinite sum discounted at the rate \(\beta \gamma\) summarizing the history in which re-optimizations never occur. The first part is the period loss function. The second part indicates the value the policymaker obtains if a re-optimization occurs in the next period. The sequence of constraints (9) corresponds to the structural equations (7), with the only exception that expectations of future variables are expressed as the weighted average between two terms: the allocations prevailing when previous plans are honored \((x_{t+1})\), and those prevailing when a re-optimization occurs \((x_{t+1}^{reop})\). This reflects the fact that private agents are aware of the possibility of policy re-optimizations, and take this possibility into account when forming their expectations. The solution uses the concept of a Markov-Perfect equilibrium and can be shown to be of the form\(^8\)

\[
\xi_t = F_s \xi_{t-1} + Gv_t \tag{10}
\]

where \(\xi_t = [x_t, \lambda_t]'\) and \(\lambda_t\) is a vector of Lagrange multipliers attached to the constraints (9). In particular, the Lagrange multipliers \(\lambda_{t-1}\) contain a linear combination of past shocks \(\{v_{t-1}, v_{t-2}, \ldots, v_{-1}\}\), summarizing the commitments made by the central bank before period \(t\). Therefore, the effects of policy re-optimizations can be described by the state dependent matrices where a re-optimization involves setting to zero the column of \(F\) corresponding to

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\(^8\)See Debortoli and Nunes (2010) and Debortoli et al. (2014) for a detailed discussion.
the Lagrange multipliers

\[ F(s_t=1) = \begin{bmatrix} F^{xx} & F^{x\lambda} \\ F^{\lambda x} & F^{\lambda\lambda} \end{bmatrix} \quad F(s_t=0) = \begin{bmatrix} F^{xx} & 0 \\ F^{\lambda x} & 0 \end{bmatrix}. \] (11)

2.3 Bond pricing

We now derive bond prices in a framework where the term structure of interest rates reflects the dynamic properties of the representative consumer’s elasticity of inter-temporal substitution (or stochastic discount factor), \( M_{t+1} \). In particular, let \( i_{n,t} \) denote the (continuously compounded) interest rate at time \( t \) of a \( n \)-period zero-coupon bond, and \( m_{t+1} = \ln(M_{t+1}) \). We have from the first order condition of inter-temporal utility maximization,

\[ e^{-ni_{n,t}} = E_t \left( e^{\sum_{\tau=1}^{n} m_{t+\tau}} \right) \] (12)

Under the assumption of a stationary joint log-normal distribution, it then follows:

\[ i_{1,t} = -E_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1}) \] (13)

\[ ni_{n,t} = -E_t \left( \sum_{\tau=1}^{n} m_{t+\tau} \right) - \frac{1}{2} Var_t \left( \sum_{\tau=1}^{n} m_{t+\tau} \right) \] (14)

We can use the above equations to decompose the yield spread, \( i_{n,t} - i_{1,t} \) into three parts,

\[ i_{n,t} - i_{1,t} = \sum_{\tau} \frac{E_t(-m_{t+\tau})}{n} - E_t(-m_{t+1}) \]
\[ - \frac{1}{2} \left[ \frac{\sum_{\tau} Var_t(m_{t+\tau})}{n} - Var_t(m_{t+1}) \right] \]
\[ - \frac{1}{2} \sum_{\tau_1 \neq \tau_2} Cov_t(m_{t+\tau_1}, m_{t+\tau_2}) \] (15)

The first term in the expression above captures the expectation component which implies that part of of the long-term interest rate, \( i_{n,t} \) is determined by the expectation of the short-term interest rate that will prevail over the life of the long-term bond. The second term is due to Jensen’s inequality. The third term is risk compensation (or term premium) for holding the long-term bond. The term premium depends critically on auto-correlations of the stochastic discount factor.\(^9\) A positively (negatively) auto-correlated stochastic discount

\(^9\)See Piazzesi and Schneider (2006) for more explanations regarding nominal and real bonds.
factor implies a negative (positive) term premium and hence a downward-sloping (upward sloping) yield curve. The intuition is as follows. If today’s bad news about growth is expected to be followed by further bad news in the future, a bond that promises a fixed payoff in the future will see its value increase today (as interest rate decreases). This create a positive co-variance between bond return and investor’s marginal utility today and hence a negative risk premium. On the contrary, if today’s bad news about growth is expected to be followed by good news in the future, a bond that promises a fixed payoff in the future will see its value decrease today (as interest rates increase). This creates a negative co-variance between bond return and investor’s marginal utility and hence a positive risk premium.

We will use this intuition to explain how the degree of monetary policy credibility affects the term structure of interest rates below. We start by considering the same simple model laid out in Section 2.2 above. We assume a power utility function that is consistent with the dynamic IS curve in (3). The stochastic discount factor for bond pricing can be written as (ignoring any constant term):

$$m_{t+1} = -\sigma (y_{t+1} - y_t) - \pi_{t+1}$$  \hspace{1cm} (16)

where $\sigma > 0$ is the coefficient of relative risk aversion. Given the solution to the optimal policy problem in (10), we can express the stochastic discount factor in general as:

$$m_{t+1} = -\lambda_0 - \lambda_1' \xi_{t+1} - \lambda_2' \xi_t$$  \hspace{1cm} (17)

where $\lambda_1$ and $\lambda_2$ load up relevant state variables respectively according to the specification of the utility function in (16) and the optimal policy solution in (10). Notice that in (10), $\xi_t$ has regime-switching dynamics governed by the variable $s_t$. We will assume that the structural shocks in (10) are normally distributed ($\nu_t \sim N(0, Q)$) and uncorrelated with each other. We use a log-linear approximation similar to the one in Bansal and Zhou (2002) and others to obtain an analytical solution for the term structure of interest rates under regime switching. This allows us to solve for the term structure of interest rates in closed form. Let $P_{n,t}$ denote the price of a $n$-period zero-coupon bond at time $t$. It then follows that, for $n \geq 0,$

$$P_{n,t} = e^{-\Lambda_n - B_n \xi_t}$$  \hspace{1cm} (18)
The coefficients $A_n$ and $B_n$ are given recursively by the following equations

\begin{align*}
A_n &= A_{n-1} + \lambda_0 - \frac{1}{2} (\lambda_1 + B_{n-1})' GQG' (\lambda_1 + B_{n-1}) \\
B_n &= \bar{F}' B_{n-1} + (\lambda_2 + \bar{F}' \lambda_1)
\end{align*}

(19)

(20)

where

$$\bar{F} = \gamma F_{(s_t=1)} + (1 - \gamma) F_{(s_t=0)}$$

(21)

and $A_0 = B_0 = 0$.

For $n \geq 1$, interest rate, $i_{n,t}$, is given by

$$i_{n,t} = \frac{A_n}{n} + \frac{B_n}{n} \xi_t$$

(22)

Notice that, because $s_t$ is i.i.d., the coefficients $A_n$ and $B_n$ don’t depend on the policy regime $s_t$ even though $s_t$ affects the persistence of the state variable $\xi_t$. To predict $F(s_{t+1})$ at time $t$ investors simply use the expected value $\bar{F}$, which is time-invariant. This has an important implication for the bond risk premium which we define as the expected excess holding period return of a long-term bond.

$$rp_{n,t} = E_t(\log P_{n-1,t+1} - \log P_{n,t}) - i_{1,t}$$

$$= \frac{1}{2} \lambda_1' GQG' \lambda_1 - \frac{1}{2} (\lambda_1 + B_{n-1})' GQG' (\lambda_1 + B_{n-1})$$

$$= -\lambda_1' GQG' B_{n-1} - \frac{1}{2} B_{n-1}' GQG' B_{n-1}$$

(23)

The second term in the risk premium expression is simply due to Jensen’s inequality. The first-term is the negative covariance of bond returns and the stochastic discount factor (under the macroeconomic shocks $v_{t+1}$). Since both $\lambda_1$ and $B_n$ are constant in our model, the covariance is constant and so is the risk premium. Relaxing the i.i.d. assumption about the transition matrix governing the re-optimization shock will generate a time-varying risk premium. However this approach makes the estimation strategy intractable and we leave it for future research. A time-varying risk premium can also be introduced into our model in ways that are popular in the literature. One convenient option is through stochastic volatility in $v_{t+1}$, as in Bikbov and Chernov (2013), Chib et al. (2012) and Amisano and Tristani (2009). In that case, $Q$ will be a function of a latent state variable. Alternatively, assuming that the market price of risk, $\lambda_1$, is subject to an exogenous preference shock,
Dew-Becker (2014) is able to generate a time-varying risk premium. Since our focus here is to understand the effect of credibility and re-optimization shocks, we abstract from these complications in our model.

2.4 Degree of Credibility and the Yield Curve

In this section we explore the effect of the degree of credibility on the term structure in the simple model outlined above.

First, consider the case of discretionary policy (i.e. \( \gamma = 0 \)). Recall that the first order condition implies that inflation responds to the level of the output gap,

\[
\pi_t^d = -\frac{1}{\theta} y_t^d
\]

Plugging this into the the stochastic discount factor (equation 16), we have:

\[
m_{t+1} = -\sigma (y_{t+1} - y_t) - \pi_{t+1} = -\left( \sigma - \frac{1}{\theta} \right) y_{t+1}^d + \sigma y_t^d
\]

where \( y_{t+1}^d = -\frac{\theta}{1+\kappa} u_{t+1} \equiv \chi_t u_{t+1} \). We can show that this implies

\[
\text{Cov}(m_{t+1}^d, m_{t+n}^d) = \begin{cases} 
\frac{(1-\sigma\theta)\sigma\theta}{(1+\kappa\theta)^2} \sigma_u^2, & \text{if } n = 2 \\
0, & \text{otherwise}
\end{cases}
\]

Thus the sign of the autocorrelation of the stochastic discount factor and hence the sign of the risk-premium depends on the weight on inflation in the loss function and the risk aversion parameter. As long as \( \sigma \theta > 1 \), \( m_{t+1}^d \) is serially negatively correlated and the yield curve slopes upward on average (ignoring the Jensen’s inequality term). The intuition is as follows. Under discretion, in response to a cost-push shock, output declines and inflation increases. Since the shock is i.i.d., expected output growth increases as the level of output moves back to its steady state level. Higher expected growth leads to a higher interest rate and hence lower bond prices (and thus lower bond returns). The sign of the risk-premium depends on the covariance between the bond return and the nominal stochastic discount factor. The direction of the movement in the nominal stochastic discount factor depends on the relative

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10 We will assume that the natural rate of output is constant so that changes in output are the same as changes in the output gap.

11 Recall from figure 1 that under discretion inflation is an i.i.d. process and expected inflation is zero.
magnitude of $\sigma$ and $\theta$. If both $\sigma$ and $\theta$ are large ($\sigma\theta > 1$), investors are very concerned with decreases in output (or consumption) and policy makers are very concerned with increases in inflation. As a result, in response to a cost-push shock, there will be relatively small increase in inflation, and given a large value of the risk-aversion coefficient, the nominal stochastic discount factor (or inflation-adjusted marginal utility) increases. On the other hand, if both $\sigma$ and $\theta$ are small ($\sigma\theta < 1$), the opposite is true, the nominal stochastic discount factor (or inflation-adjusted marginal utility) decreases in response to a cost-push shock.

In contrast, under a full-commitment policy, inflation responds to changes in output,

$$\pi_t^c = -\frac{1}{\theta}(y_t^c - y_{t-1}^c)$$

and hence,

$$m_{t+1}^c = -(\sigma - \frac{1}{\theta})(y_{t+1}^c - y_t^c)$$

where $y_{t+1}^c = \chi^c(y_{t+1}^c + \chi^c u_{t+1}$ and $\chi^c < 1$. We can show that this implies

$$\text{Cov}(m_{t+1}^c, m_{t+n}^c) = -\left(\sigma - \frac{1}{\theta}\right)^2 \chi^c n^{-2} \sigma_u^2 \frac{(1 - \chi_u^c)^2}{1 - (\chi_u^c)^2}\chi^c$$

Thus regardless of the value of $\theta$ and $\sigma$, the stochastic discount factor is serially negatively correlated. Bond risk premium is always positive under a full-commitment policy in this simple model and the yield curve slopes upward. The reason is that, under full commitment, the nominal interest rate can either increase or decrease, but it always moves in the same direction as the nominal stochastic discount factor in response to a cost-push shock regardless of the relative values of $\sigma$ and $\theta$. With a full commitment technology, the policy maker can promise negative inflation (or deflation) in the future at the impact of a cost push shock. If the nominal stochastic discount increases as in the case of $\sigma\theta > 1$, the expected deflation will be smaller than expected output growth, the nominal interest rate also increases. If the nominal stochastic discount factor decreases as in the case of $\sigma\theta < 1$, the expected deflation will be larger than the expected output growth, the nominal interest rate decreases as well.

Finally, as pointed out by Palomino (2012), $y_{t+1}$ has a bigger exposure to the cost-push shock in the discretionary-policy regime than in the full-commitment policy regime. From (24) and (27), we have:

$$m_{t+1}^i - E_t(m_{t+1}^i) = -(\sigma - 1/\theta)\chi_u^i u_{t+1}$$
where $i = \{d, c\}$. The absolute value of market price of risk under a discretionary policy, $|\sigma - 1/\theta_0^d|$, will be larger than that under a full-commitment policy, $|\sigma - 1/\theta_0^c|$. This has a direct impact on the magnitude of bond risk premiums.

Under loose-commitment, the first order condition implies

$$\pi_{lc}^{ic} = \begin{cases} 
-\frac{1}{\theta} y_{lc}^t, & \text{if } s_t = 0 \\
-\frac{1}{\theta} (y_{lc}^t - y_{lc}^{t-1}), & \text{if } s_t = 1 
\end{cases}$$

In this case the auto-covariance of the stochastic discount factor does not have a convenient analytical solution. To better understand the effect of the degree of credibility ($\gamma$), in figure 3 we plot the model implied yield curve for different values of $\gamma$. In panel (a) we consider a calibration where $\sigma\theta < 1$. As pointed above, under such a calibration the yield curve slope is negative under discretion (blue line) and it is positive under full commitment (yellow line). In the loose commitment setting, the slope is negative for low values of $\gamma$ and it becomes less negative as $\gamma$ is increased before finally becoming positive. In panel (b) we show the yield curve with a calibration where $\sigma\theta > 1$. In this case the slope is non-negative for all values of $\gamma$ and we see the same pattern that the slope increases with $\gamma$. The degree of monetary policy’s credibility shapes expectations about future inflation, and hence has a key effect on the co-movement between the nominal interest rate and the stochastic discount factor. This effect is reflected in the shape of the term structure of interest rates. In particular, as $\gamma$ increases, the monetary policy is more likely to remain on its promised course. Expected inflation then tends to move in the direction that produces a positive (negative) co-variance between interest rate (bond return) and the stochastic factor, and hence an upward sloping yield curve as explained above. The next figure shows how the unconditional standard deviation of the yield curve depends on $\gamma$. Again, panel (a) shows the case where $\sigma\theta < 1$ and panel (b) shows $\sigma\theta > 1$. The short end of the yield curve is always more volatile under discretion relative to full commitment regardless of the preference and policy parameters. This is because, by promising lower (or negative) inflation in the future with full commitment, the central bank doesn’t need to raise interest rates as much as it does under discretion in response to a cost push shock. We notice that as $\gamma$ increases agents have a higher confidence that the central bank will continue proposed plans and thus they adjust their current inflation expectations accordingly. Thus the volatility of the short end of the yield curve decreases with $\gamma$. For the second calibration this relationship is true for yields

\footnote{Notice that when $\sigma\theta > 1$ the yield curve is flat instead of sloping upward under discretion ($\gamma = 0$) because a negative Jensen’s inequality term offsets a positive bond risk premium at each maturity.}
of all maturities. But in the calibration in panel (a) we see a non-monotonic relationship where the volatility of long-term rates is the highest for $\gamma = 1$. This is because the volatility of a long-term interest rate depends not only on the volatility of the short-term rate, but also on the persistence of the short-term interest rate. Other things being equal, a more persistent short-term interest rate generates more volatile long-term rates. The short-term interest rate is the most persistent under full-commitment.

Overall figures 3 and 4 show that the loose commitment framework is quite flexible and depending on the probability of commitment ($\gamma$) can generate a variety of different properties for slope and standard deviation of the term structure. In addition to the effect of $\gamma$ the loose commitment framework also has important implications for the dynamic behavior of yields as governed by re-optimization shocks.

2.5 Re-optimization Shocks and the Yield Curve

In this section we analyze the effect of a re-optimization shock on the term structure in the simple model. As mentioned above, the impulse response to a re-optimization shock is history dependent. The re-optimization shock involves reneging on past promises which are captured by the Lagrange multipliers. Thus the effect of the re-optimization shock is to set the lagged Lagrange multipliers to zero. As a special case, if a re-optimization shock occurs in the steady state it will have no effect as the Lagrange multipliers are already zero, while a re-optimization occurring immediately after a cost-push shock that creates a tradeoff for the central bank can have big effects.

In the same vein as figures 1 and 2, figure 5 shows the effect of a cost-push shock happening at time period 1, followed by a re-optimization shock occurring in period 3. $\gamma$ is set to 0.5. The discretion and full commitment paths (which are not affected by the re-optimization shock) are plotted for comparison. The solid thin blue line shows that in response to a cost-push shock the central bank raises the short rate and promises to gradually decrease it to zero over time. When the re-optimization occurs, the central bank sets the short rate to zero to bring inflation back to zero immediately. Since the cost-push shock is i.i.d. all the long-term yields move to zero instantly as well. One way to gauge the effect of the re-optimization shock is to compare the value of the yields under a re-optimization with the value if no re-optimization shock had occurred. In figure 5, this is the difference between the thin blue line and the thick blue line with the crosses. To better understand
the effect on the yield curve, we construct three term structure factors that are commonly studied in the literature: level, slope and curvature. The level is defined as just the 3 month rate, the slope as (10 year - 3 month) and the curvature as (10 year + 6 month - 2*3 year).

Figure 6 plots the effect of the re-optimization shock on these three factors. We see that a re-optimization shock in this model increases the slope of the term structure while lowering the level and the curvature. Notice that the x-axis in the graph represents time and starts when the re-optimization shock hits, i.e. period 3. In this simple model, the effect of the cost-push shock monotonically decreases with maturity, thus when a re-optimization causes yields to be set to their steady-state value the fall is biggest at the short end of the yield curve and the effect diminishes with maturity. This causes the yield curve to become more steep and less curved as a result. This simple example illustrates that, depending on the history of past economic shocks, the re-optimization shock can generate rich dynamic responses of interest rates. It not only affects the short-term interest rate, but can also have profound effects on the entire yield curve.

Using a simple New Keynesian model we have shown in sections 2.4 and 2.5 that both the degree of credibility \((\gamma)\) and the timing of the re-optimization shocks can have important implications for the yield curve. Next we conduct an empirical analysis to quantify these effects for post Great Moderation US data. We expand the simple macro model and use a medium scale DSGE model that is known to fit the US macro data well.

3 Medium-Scale DSGE Model

The DSGE model we use is from Smets and Wouters (2007) (SW henceforth) and is based on earlier work by Christiano et al. (2005) among others. This model has been shown to fit the macro data well and is competitive with reduced form vector auto regressions in terms of forecasting performance. The model includes monopolistic competition in the goods and labor market, nominal frictions in the form of sticky price and wage settings, allowing for dynamic inflation indexation. It also features several real rigidities – habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. For further details of the model we refer the reader to SW.\(^{13}\)

We depart from the SW formulation in the specification of monetary policy. In SW,\(^{13}\)In the online appendix accompanying the SW paper, available at https://www.aeaweb.org/aer/data/june07/20041254_app.pdf, a detailed derivation of the model’s equations is provided.
monetary policy is described by an interest rate policy rule, while in this paper the central bank is modeled as minimizing a loss function under the loose commitment framework as described in section 2.2. The period loss function that we use for the empirical results is the following:

\[ x_t' W x_t \equiv \pi_t^2 + w_y \tilde{y}_t^2 + w_r (r_t - r_{t-1})^2 \]  

(29)

The weight on inflation (\(\pi_t\)) is normalized to one so that \(w_y\) and \(w_r\) represent the weights on output gap (\(\tilde{y}_t\)) and the nominal interest rate (\(r_t\)), relative to inflation. \(\pi_t\) represents the deviation of inflation from the steady state, implying that the inflation target is the steady state level of inflation \(\bar{\pi}\), which will be estimated. The target for output is the “natural” counterpart, defined as the level of output that would prevail in the absence of nominal rigidities and markup shocks. This formulation is consistent with the natural rate hypothesis, i.e. that monetary policy cannot systematically affect average output. The last term in the loss function \((w_r (r_t - r_{t-1})^2)\) indicates the central bank’s preference for interest rate smoothing, see Coibion and Gorodnichenko (2012) for a detailed discussion.

3.1 Estimation

In the estimation we use the same quarterly US time series as SW, except we replace the fed funds rate with the 3 month Treasury bill. The macro series are as follows: the log difference of real GDP, real consumption, real investment, the real wage, log hours worked and the log difference of the GDP deflator. In addition to the 3 month rate, we use 5 more yields: 6 month, 1 year, 3 year, 6 year and 10 year. For the yields we use the data from Gürkaynak et al. (2007). In the SW setup, there is a monetary policy shock in the interest rate rule. We also add a monetary policy shock such that

\[ r_{t}^{\text{obs}} = r_t + e_t^r \]  

(30)

\[ e_t^r = \rho_r e_{t-1}^r + \eta_t^r \]  

(31)
with the assumption that $\eta^r_t \sim N(0, \sigma^2_r)$. For estimation we can write the system as the following state space model.

$$\xi_t = F_s \xi_{t-1} + Gv_t$$

$$Y_{t}^{obs} = A + H\xi_t + w_t$$

$$v_t \sim N(0, Q)$$

$$w_t \sim N(0, R)$$

$$P = \begin{bmatrix} Pr(s_t = 1|s_{t-1} = 1) & Pr(s_t = 0|s_{t-1} = 1) \\ Pr(s_t = 1|s_{t-1} = 0) & Pr(s_t = 0|s_{t-1} = 0) \end{bmatrix} = \begin{bmatrix} \gamma & 1 - \gamma \\ \gamma & 1 - \gamma \end{bmatrix}$$

The state equation (32) corresponds to the macro dynamics governed by optimal policy under loose commitment (described earlier in equation (10)). Due to the possibility of re-optimizations, the parameter matrix $F_s$ depends on the regime switching variable $s_t$ whose transition matrix is governed by equation (36). The errors $v_t$ include all the structural shocks from the SW model, including the monetary policy shock. The observed macro variables and the yield data are stacked in $Y^{obs}_t$ and the observation equation (33) relates them to the state variables $\xi_t$. For the macro variables the matrix $H$ just picks out the corresponding variables from the state vector $\xi_t$, while the elements in $A$ capture the steady state trends. For the yield data in $Y^{obs}_t$, the corresponding elements of $A$ and $H$ capture the term structure relationship derived in equation (22). Finally, we add an i.i.d. measurement error to each of the yields in the observation equation, given by $w_t$.

With regards to the yields, our empirical specification has two more points worth emphasizing. In the model, the central bank sets the short rate, here the 3 month T-bill rate, as dictated by minimization of the loss function. Thus in our initialization of the bond pricing recursion (equations (19) and (20)) we start with $n = 1$ instead of $n = 0$ and set $A_n(1)$ equal to the constant in the 3 month T-bill rate equation in the observation equation and we set the corresponding element of $B_n(1) = 1$ which picks out the short rate from the state vector $\xi_t$. Additionally, to help the model fit the average level of the term structure we treat $\lambda_0$ in equation (19) as a free parameter to be estimated. When we tried to let $\lambda_0$ be consistent with the utility function, we found that the model underestimated the level of the yield curve but the model’s implied slope and dynamics for the yield curve were similar to our baseline results.

The estimation algorithm used here is similar to the one outlined in Debortoli and
Lakdawala (forthcoming). To summarize, the regime-switching model requires using the Kim (1994) approximation that combines the Hamilton (1989) filter and the Kalman filter to evaluate the likelihood function, see Kim and Nelson (1999) for details. This likelihood function is combined with the prior to form the posterior distribution. A Metropolis-Hastings algorithm is used to sample from the posterior distribution.

3.2 Estimates from DSGE Model

Table 1 shows the posterior mean of the estimates of the structural parameters, along with the 5th and 95th percentile values. In Table 2 the estimates of the parameters of the shock processes are shown. Overall the parameter estimates are similar to SW with a few exceptions.\(^{14}\) The estimates of the utility function are somewhat different. Notably, the inverse of the elasticity of intertemporal substitution is estimated to higher here \((\sigma_c = 2.16)\) compared to SW \((\sigma_c = 1.47)\), while habit persistence parameter is lower in our estimates. The persistence of the risk-preference shock (referred to by SW as the “risk-premium” shock) is higher in our estimates. This shock can be broadly thought of as a demand shock and has effects which are similar to a net-worth shock in models that have an external finance premium. Our estimates of the capital capacity utilization and capital adjustment costs are also slightly higher relative to SW. The weight on interest rate smoothing is higher here compared to Debortoli and Lakdawala (forthcoming), most likely reflecting our different sample size which excludes the pre 1980s data which had larger movements in the short-term interest rates. Finally there are some differences in the parameters of the price-markup shock. See Debortoli and Lakdawala (forthcoming) and Justiniano and Primiceri (2008) for a detailed discussion.

The probability of commitment \((\gamma)\) is estimated to be 0.86 with high precision. The use of quarterly data implies that the Federal Reserve is expected to re-optimize plans roughly every seven quarters. This is similar to the finding in Debortoli and Lakdawala (forthcoming) where they also allow for regime-switching in the shock variances. In the loose commitment setting, both the agents and the central bank have full information, including which regime is prevalent at any given time. This information about the prevailing regime is not observable to the econometrician. Nevertheless, we can back out an estimate of this by looking at the smoothed probability of re-optimization. This can be interpreted as the probability of a

\(^{14}\)Given our data sample, we compare our estimates to the second sub-sample of the results reported in Table 5 of SW.
re-optimization having occurred on any given date conditional on observing all the data. We can use this probability to try to characterize when the re-optimization episodes were likely to have occurred in the historical data. To this end, in figure 7 we plot the smoothed probability of re-optimization. If we consider a probability greater than 0.5 as a likely re-optimization episode, we notice a few prominent episodes. In the early 1980s there are two episodes which could potentially correspond to the end of the experiment of reserves targeting that was undertaken in the early Volcker years. In the early and mid-1990s we see a handful of episodes that could be related to the policy change that the Federal Reserve undertook of releasing its target for the federal funds rate, along with statements of the committee’s opinion on the direction of the economy. Finally there are re-optimization episodes that correspond to attacks of 9/11 in 2001 and the onset of the financial crisis at the end of 2007. Notably, three long periods (mid 1980s to early 1990s, late 1990s and early 2000s till the start of the financial crisis) stand out for not having any likely re-optimization episodes. In section 3.4 we discuss how the re-optimization episodes affect the yield curve, but we first discuss the model’s performance in terms of the term structure.

To evaluate how well our model fits the term structure, we first plot the yield data with the corresponding model implied fit in figure 8. Overall, the model does a reasonable job of fitting the yield data, especially at the short end of the yield curve. The estimated standard deviations of the measurement errors range from 42 basis points for the 6 month yield to 77 basis points for the 10 year at an annualized rate. In comparison, the unconditional standard deviation of the yields in our sample is over 200 basis points for all the yields. However, the model’s fit is not competitive with the best term structure factor models. We believe that there are a few natural extensions that can help improve the fit in our setting. First, De Graeve et al. (2009) use the SW model (with an interest rate rule) and find that a time-varying inflation target significantly improves the fit of the longer yields. For example, the shock to the inflation target in their paper explains more than 90% of the variation in the 10 year yield. Rudebusch and Swanson (2012) also acknowledge the importance of the time-varying inflation target in matching yield data. Second, introduction of regime-switching volatility in the variances of the shocks can help the model generate a time-varying term premium which results in higher volatility at the long end of the yield curve. Bikbov and Chernov (2013), Chib et al. (2012) and Amisano and Tristani (2009) use this approach in a simpler DSGE model. Third, a time-varying term premium could be introduced by allowing parameters of the utility function to change. Dew-Becker (2014) uses this approach by modeling time variation in the risk-aversion parameter. Finally, rather than assuming an
i.i.d. re-optimization shock, we can introduce a Markov structure in the transition matrix governing the re-optimization shocks. With this state-dependence in the re-optimizations, the factor loading of bond prices in the term structure solution will be regime-dependent and time-varying, which will generate time-varying bond risk premia. These approaches could be adopted in our current framework. However, since our paper represents the first effort to study the empirical effects on the term structure of Federal Reserve credibility and re-optimization shocks, we leave these extensions for future research.\footnote{Additionally, Rudebusch and Swanson (2012) show that using Epstein-Zin preferences in a DSGE model can help drastically improve the fit of the model. But that would require a higher-order approximation to solve the model which would make the computation of optimal policy and estimation intractable.}

Next, we discuss the contribution of the various macroeconomic shocks in explaining the variation in the yield curve. In table 3 we present the forecast error variance decomposition, where we weight the the coefficients under each regime by the unconditional probability of being in that regime, i.e. we use $\gamma F_{s_t=1} + (1-\gamma)F_{s_t=0}$. For the purpose of this exercise, following SW we lump together government spending, investment and risk preference shocks into a unified demand shock. Additionally we lump together the price markup and wage markup shocks into a unified markup shock. A few interesting results stand out. Markup, demand and monetary policy shocks are important drivers of yield curve while technology shocks contribute very little to the variation in either short or long-term yields. Demand shocks explain a significant variation of the yield curve fluctuation at all maturities but the effect is hump shaped in maturity with the biggest effect on medium term yields, explaining 69\% of the variation in the 3 year yield. For the short to medium term yields (6 month, 1 year and 3 year), the markup shock explains very little of the forecast error variance at the 1 and 5 quarter horizon, whereas for the longer term yields it accounts for a much bigger share. For all yields, as the forecast horizon increases, a larger share is attributable to the mark-up shock. The markup shock explains more than three-quarters of the variation in the 10 year yield at the 20 quarter horizon. At the short end of the yield curve, the monetary policy shock contributes a larger share to the variance but its contribution diminishes with the forecast horizon. The contribution of structural shocks to historical U.S. data will be important in determining the effects of re-optimization shocks, as we will discuss in section 3.4.
3.3 Counterfactual Analysis

Next we turn our attention to a counterfactual simulation. We would like to answer the following question: How would the historical path of yields have been different if the Federal Reserve had acted under full commitment or discretion? We first back out the structural shocks for our model from the benchmark estimates. Next, fixing these shocks we simulate the path of interest rates while changing the probability of commitment to 0 (for discretion) and 1 (for full commitment). These simulations are presented in figure 9, where the red, blue and black lines represent the data, full commitment and discretion cases respectively. This graph shows that neither full commitment nor discretion can capture the yield data completely. Sometimes the data is closer to full commitment (especially in the 2000s for longer yields), whereas sometimes the data is closer to discretion (for example, short to mid-term yields in the middle of the 1990s). This suggests that the more general loose commitment framework is important to correctly characterize the movement in yields. We also notice a consistent pattern that the yields would have been lower on average under discretion relative to full commitment. Another way to see this is to look at the model implied steady state level of the yields under discretion and full commitment, and compare them to the data and those under loose commitment. This is shown in table 4. We can see that assuming a discretion policy regime would produce the worst fit of the yield curve. The steady state yields under discretion for all maturities are much lower than the data. They are also lower than those under full commitment and loose commitment. Under discretion, the yield spread between 10 year bond and 3 month Treasury bill is about 1.05%, while in the data it is about 1.86%. Full commitment produces a better fit of the average yield at shorter maturities, with implied 6 month to 3 year maturities actually closer to the data compared to those under loose commitment. But full commitment produces a much flatter yield curve compared to the data. The yield spread between 10-year bond and 3 month bill under full commitment is only 0.96%, about half of that in the data. The steady-state yield spread under loose commitment, however, is 1.92%, which is much closer to the data.

3.4 Effect of Re-optimization Shocks

What is the effect of a re-optimization shock on the yield curve? In the loose commitment framework, the effects of a re-optimization depend on the history of past shocks, as discussed in section 2. Figures 10, 11 and 12 illustrate this phenomenon showing the impulse
responses to a price-markup shock, technology and risk-preference shock respectively. The solid blue line shows the path under the assumption that a re-optimization never occurs (even though agents expect it to occur with probability 0.86). The line with dots refers to the scenario where a re-optimization occurs once after 7 quarters, but not after that. The difference between the two lines thus measures the effects of a policy re-optimization that occurs in period $t = 7$. The figure also shows the impulse responses under discretion and commitment which are obtained by just setting $\gamma = 0$ and $\gamma = 1$ but keeping the rest of the parameters fixed at the estimated values. The effect of a re-optimization that is preceded by a price-markup shock (figure 10) or risk-preference shock (figure 12), is to make yields lower relative to the case where no re-optimization occurs. The intuition is similar to the one in the simple model of section 2, where the central bank would like to bring inflation and interest rates to their steady-state levels sooner. On the other hand, a re-optimization shock that occurs after a technology shock, makes the central bank want to set interest rates higher than promised, as can be seen in figure 11. The sign of the effect is similar for yields of all maturities but the magnitude varies. To better understand the differential effects on the yield curve, in figure 13 we plot the effects of a re-optimization shock on the three factors: level, slope and curvature. We define these in the same way as in section 2.5. The level factor as just the 3 month rate, the slope factor as 10 year - 3 month and the curvature factor as 10 year + 6 month - 2*(3 year). Specifically, we plot the difference between the thin blue line and the thick blue line with the dots (from figures 10-12). Notice that the x-axis in the graph represents time and starts when the re-optimization shock hits, i.e. period 7. Each row shows the effect after a specific structural shock. Re-optimizations following risk-preference, investment and markup shocks have sizeable effects on the three factors, while re-optimizations following monetary policy, technology and government spending shocks have smaller effects. The response of the level and slope have opposite signs. For instance, the level falls and slope rises after markup shocks. There emerges an interesting pattern in the dynamics responses of the three factors. The peak effect on the curvature is contemporaneous, i.e. in the same period as the re-optimization. On the other hand the level and slope peak a few quarters after the re-optimization. It is important to note that in addition to the effects of a re-optimization depending on which shocks have preceded it, the timing of those shocks matter as well. As an extreme example, when the economy is in the steady state, a re-optimization will have no effect. On the other hand if a structural shock has hit the economy, and a few quarters have passed (which allow the effects of the shock to peak), then a re-optimization shock can have big effects.
Next, using our model and estimated structural shocks, we try to quantify the effects of re-optimization shocks in the historical U.S. data. We answer the following hypothetical question: Given the past history of shocks in the U.S. data (implied by our model), what would be the effect if a re-optimization occurred in every period? To this end, we plot the difference between the reaction of yields if a re-optimization occurs relative to the case where one does not occur, conditional on the estimated structural shocks. Using the model’s equations, this can be written as $[i_{n,t}|s_t = 0, \xi_{t-1}] - [i_{n,t}|s_t = 1, \xi_{t-1}]$, where $i_{n,t}$ refers to the yield of maturity $n$ in percentage points. $\xi_{t-1}$ represent the smoothed state variables that capture the full history of shocks up to time $t-1$. Figure 14 shows this difference for the 6 yields. The dashed black vertical lines represent time periods when the probability of re-optimization from figure 7 is greater than 0.5. As observed in the impulse response figures, we notice that re-optimizations can act as both contractionary and expansionary shocks. For example, in the re-optimization that is likely to have occurred in 1985:Q3 the effect was to lower yields significantly relative to continuing plans. While the re-optimization episode of 1994:Q4 resulted in higher yields. This reinforces the notion that the effect of a re-optimization is crucially dependent on past historical shocks. While the effect of the re-optimization episodes is small on the 3 month yield, it does not increase monotonically with maturity. The largest effect of the re-optimization shocks is on the 3 year followed by the 1 year. The effect of the 10 year yield is very similar to the effect on the 6 month.

To explore this non-monotonic effect of re-optimizations in more detail, we also consider the the response of the constructed term structure factors. Additionally, note that figure 14 plots only the contemporaneous response of yields to a re-optimization shock. But the analysis in figure 13 suggests that there can be significant lags in the effects. Thus in figure 15 we consider the “medium-term” and “long-term” effects in addition to the contemporaneous effect. Specifically, we plot $[f_{k,t+j-1}|s_t = 0, \xi_{t-1}] - [f_{k,t+j-1}|s_t = 1, \xi_{t-1}]$ where $f_{k,t+j}$ is the response of the $k$th term structure factor in time period $t+j$. The blue line is the contemporaneous response ($j = 1$ quarter), the green line is the “medium-term” response ($j = 8$ quarters) and the red line is the “long-run” response ($j = 40$) quarters. Again, the dashed black vertical lines show time periods when the probability of re-optimization from figure 7 is greater than 0.5. The graph suggests that four re-optimization episodes have had the biggest effect on the yield curve: 1985:Q3, 1994:Q4, 2001:Q4 and 2007:Q4, while

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16To clarify, the medium and long term response is calculated in the following way. We assume a re-optimization occurs only once in time period $t$ and then trace out the response of the yields. From this response we subtract the behavior of the yields in the scenario where a re-optimization does not occur. The other shocks are set to zero from period $t$ onwards.
the other re-optimization episodes have had smaller effects. Interestingly, re-optimization episodes before the 1990s contributed to lowering the level while increasing the slope and curvature. Since the 1990s that pattern has reversed. Consistent with the analysis in figure 13, for the level and slope the biggest effects of the re-optimization shock occur in the medium term. The long-term and contemporaneous effects are much smaller. On the other hand, the biggest effect on the curvature factor is contemporaneous with smaller effects in the medium and long run. Finally for comparison, the standard deviation of the level, slope and curvature factors calculated from the data is 2.22, 1.23 and 0.80 percentage points respectively. This suggests that relative to the overall movement in the factors, re-optimization shocks can account for a non-negligible proportion of the variation.

4 Conclusion

The Federal Reserve is keenly interested in understanding how changes in its policy instruments translate into changes in the economy. This transmission mechanism works through the effect of the policy instrument (typically the short interest rate) on the long rates. Instead of using the standard Taylor rule setup, this paper focuses on optimal monetary policy and central bank credibility to get a deeper structural understanding of the effects of central bank actions on the term structure. In a simple model we explain the intuition behind how the flexible loose commitment framework affects the yield curve by comparing it to the commonly used discretion and full commitment cases. We highlight two features that can have important implications for the yield curve: the existing degree of credibility and the timing and frequency of re-optimization shocks.

We quantify these effects by estimating a medium-scale DSGE model where the central bank conducts optimal policy under loose commitment. This structural macro model is augmented with bond prices that are consistent with agents’ optimization decisions and the resulting system is jointly estimated using regime-switching Bayesian techniques. Consistent with earlier work, we find that the Federal Reserve is credible to some extent, but that credibility is not perfect. Moreover, neither full commitment nor discretion can do a satisfactory job of explaining term structure dynamics. Additionally, we find that re-optimization shocks can work as both expansionary and contractionary shocks in affecting yields. While these shocks affect the term structure in a variety of ways, we find the biggest contemporaneous effects on the curvature of the yield curve.
A natural extension is to allow the probability of re-optimization to be regime-dependent rather than the i.i.d. case that is used in this paper. While this would make the computation of optimal policy under loose commitment more complicated, it would have the advantage of generating a time-varying term premium where the underlying model can still be linear. Such a setup would make it feasible to conduct an empirical study where re-optimization shocks could provide a structural explanation for the change in the term premium over time.
References


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<th>Prior Distribution</th>
<th>Mean</th>
<th>Std. Dev</th>
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<th>5%</th>
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Table 2: Parameters of the Shock Processes

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Table 3: Forecast error variance decomposition

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Table 4: Model implied steady state yields

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<th>6 year</th>
<th>10 year</th>
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<td>Full Commitment</td>
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Figure 1: Impulse-response to i.i.d. cost-push shock in simple model
Figure 2: Impulse-response to i.i.d. cost-push shock in simple model

The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur even though agents expect it to occur with probability 0.5, and dashed blue line displays a re-optimization occurring in the 3rd quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Figure 3: Average yield curve in simple model

Note: The figure shows the average yield curve as a function of the probability of commitment ($\gamma$). Panel (a) uses the calibration with $\sigma \theta < 1$, while panel (b) uses a calibration with $\sigma \theta > 1$. 
Figure 4: Standard deviation of yield curve in simple model

Note: The figure shows the unconditional standard deviation of the yield curve as a function of the probability of commitment ($\gamma$). Panel (a) uses the calibration with $\sigma \theta < 1$, while panel (b) uses a calibration with $\sigma \theta > 1$.
Figure 5: Impulse-response to i.i.d. cost-push shock in simple model

Note: The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur even though agents expect it to occur with probability 0.5, and dashed blue line displays a re-optimization occurring in the 3rd quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Figure 6: Response of term structure factors to re-optimization shock in simple model

Note: The figure shows the effects of a re-optimization shock occurring after a cost-push shock. Specifically, it shows the difference between the path of the factors when a re-optimization occurs in the 3rd quarter after a cost-push shock, relative to the path when no re-optimization occurs.
Figure 7: Smoothed Probability of Re-optimization
Figure 8: Model Fit
Figure 9: Counterfactual Simulation: Yields

Note: This figure shows the counterfactual paths of yields simulated from the estimated model fixing the probability of commitment to 1 for full commitment (blue line) and 0 for discretion (black line). The red line shows the data.
Figure 10: Impulse Response: Markup Shock

Note: Impulse responses to a 1 standard deviation price markup shock under alternative commitment settings. The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur and dashed blue line displays a re-optimization occurring in the 7th quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Figure 11: Impulse Response: Technology Shock

Note: Impulse responses to a 1 standard deviation technology shock under alternative commitment settings. The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur and dashed blue line displays a re-optimization occurring in the 7th quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Note: Impulse responses to a 1 standard deviation risk-preference shock under alternative commitment settings. The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur and dashed blue line displays a re-optimization occurring in the 7th quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Note: Impulse responses to a 1 standard deviation risk-preference shock under alternative commitment settings. The blue lines indicate the responses under loose commitment; the solid one assumes that no re-optimizations occur and dashed blue line displays a re-optimization occurring in the 7th quarter, but not afterwards. The green and red line show the responses under full commitment and discretion respectively.
Note: The figure shows the effects of re-optimizations over time, measured as the difference between the value conditional on re-optimization and the value conditional on continuation of previous commitment.
Figure 15: Re-optimization effect on factors

Note: The figure shows the effects of re-optimizations over time, measured as the difference between the value conditional on re-optimization and the value conditional on continuation of previous commitment.