Modeling Monetary Policy Dynamics: A Comparison of Regime Switching and Time Varying Parameter Approaches

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Abstract

Structural VAR models have been widely used to model monetary policy dynamics. Typically, a choice is made between regime switching and time varying parameter models. In this paper we use a canonical model of monetary policy and estimate both types of time variation in monetary policy while also allowing for changing variances. The models are compared using marginal likelihood and forecasting performance. We find that for both frameworks, the best-fit model implies a specification with changes only in the variance of shocks and the implied model dynamics are similar. However, researchers using a specification with only changes in monetary policy coefficients would reach different conclusions about the dynamic behavior of monetary policy.

JEL *classification*: C32, E52, E47.

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1 Introduction

There is a large literature that studies changes in the behavior of monetary policy setting in the post-war US data. The most common way to model monetary policy, both in structural macro models and reduced form VAR models, is to use a Taylor-type interest rate rule. This is a simple linear rule where the short term interest rate is adjusted in response to some measure of inflation and economic activity. To allow for changes in the behavior of monetary policy, the parameters of this simple rule are allowed to change over time. From a modeling perspective, the econometrician has to specify some process for the time variation in the parameters. There are two common approaches that have been used in the literature. The first one, known as the regime switching (rs) approach, assumes that there are discrete changes in the parameters which are governed by a Markov switching variable. This approach has been used in VAR analysis by Sims and Zha (2006) and in structural DSGE models by Bianchi (2013) and Liu et al. (2011) among others. The other approach, known as the time varying parameter (tvp) approach, assumes that there are gradual changes in the parameters. The time variation is typically specified as an autoregressive process. This has been used in VAR analysis by Cogley and Sargent (2005) and Primiceri (2005) and in structural DSGE models by Fernandez-Villaverde et al. (2010). Sometimes, a theoretical model is used to motivate the choice between the two frameworks, but more often the choice turns out to be driven by convenience and tractability. In this paper we compare the two approaches using a structural VAR model and explore the implications for monetary policy dynamics.

Specifically, we want to study whether the choice of one method over the other yields results that lead to divergent answers to important questions related to monetary policy. First, there is a debate in the literature regarding changes in the macroeconomy since the 1980s. Was there a change in monetary policy or was it just a change in the volatility of shocks hitting the economy, or perhaps both? One approach has been to estimate models with time variation allowed in both the monetary policy parameters and the shock volatility parameters.[1] Then some form of model comparison is performed to identify the specification which best fits the data. In this paper we want to examine

[1]See Primiceri (2005) and Sargent et al. (2006) for examples that use tvp and Sims and Zha (2006) for a rs approach.

whether using a tvp or rs approach gives rise to a different conclusion. Second, if there has been a change in monetary policy, how does the choice of tvp vs. rs affect the characterization of historical changes in monetary policy? For instance, was there a big change in the reaction of the Federal Reserve to inflation when Paul Volcker was elected or was it gradual, beginning before the arrival of Volcker? Another important question is whether Federal Reserve policy in the 1970s satisfied the Taylor principle. Finally, how does this choice affect the evaluation of monetary policy's effects on the economy? A key component in this evaluation is the identification of monetary policy shocks and we investigate whether the choice of rs vs. tvp matters for the series of identified monetary shocks.

We consider a canonical three variable structural VAR with unemployment, inflation and the fed funds rate.[2] The VAR is identified using the recursive (also called triangular) identifying restriction that is most common in the monetary policy literature. Under this assumption, both inflation and unemployment react with a lag to changes in the fed funds rate while the fed funds rate is allowed to react contemporaneously to inflation and unemployment. The equation corresponding to the interest rate is interpreted as the monetary policy reaction function. Since the focus of the paper is on monetary policy, the coefficients of only this equation are allowed to change over time, either using a time varying parameter or regime switching framework. The variance of the shocks are also allowed to change over time. The tvp specification is based on the framework of Primiceri (2005) with the same prior specification.[3] The rs specification is similar to the models in Sims et al. (2008), with the main difference that we use fairly loose priors that do not put any restrictions on the time variation. This is done to keep the priors in the rs setup comparable to the tvp setup. All models are estimated using a Bayesian Markov Chain Monte Carlo algorithm.

The main results regarding the monetary policy questions are as follows. First, if a researcher were to adhere to either the tvp or rs framework and consider the best fit model within

[2]Others have studied the question of time variation in monetary policy and shock variances in DSGE type models. Bianchi (2013) and Fernandez-Villaverde et al. (2010) allow time variation in coefficients of the Taylor rule while Liu et al. (2011) model time variation in the inflation target. We choose a VAR framework as it makes the comparison between the rs and tvp frameworks more straightforward as compared to a DSGE type setting where the modeling of agents' expectations, equilibrium definitions and model solution techniques are not agreed upon in the literature.

[3]The only difference is that in this paper time variation in the coefficients is only allowed in the monetary policy equation.

that framework, the specification with changes only in the variance would be picked for both the rs and tvp specifications. Even though there are some differences in the dynamics of the estimated changes in the variances, the monetary policy shocks identified are almost identical implying very similar effects of monetary policy on the economy. Thus our results are consistent with the literature that finds that the data are more supportive of models in which the variance of shocks - rather than monetary policy coefficients - have changed over time (see Primiceri (2005), Liu et al. (2011) and Sims and Zha (2006)). But, as motivated in Benati and Surico (2009), it is also interesting to look at a framework with just changes in the monetary policy coefficients. Here we find that the dynamic patterns are different between the tvp and rs cases. Specifically, the changes in the response to inflation are more gradual in the tvp case. Additionally, the Taylor principle is satisfied for essentially the whole sample in the tvp vase. On the other hand, the rs specification implies frequent changes in the 1970s between high response to inflation that satisfies the Taylor principle and low responses that do not satisfy it. As a result the monetary policy shocks identified by the rs regime are quite different (especially in the 1970s) from the tvp case, which has important repercussions for the evaluation of monetary policy's effect on the economy. Thus our results highlight the fact that the choice of rs vs. tvp can matter a lot if the researcher does not do an exhaustive search to find the best fit model from the various available specifications.

Finally, the model comparison exercise highlights an important issue that researchers may encounter while choosing between rs and tvp frameworks. We first calculate the marginal likelihood using the approach that is most popular in the literature, i.e. the modified harmonic mean estimator, based on the work by Gelfand and Dey (1994) and Geweke (1999). The marginal likelihood calculations for the tvp case turn out to be more sensitive to the prior specification than for the rs case. This is especially true when the coefficients of the VAR are allowed to vary over time. We shed light on how the commonly used Inverse-Wishart prior contributes to this sensitivity. To further investigate this issue we conduct a recursive estimation study to evaluate the predictive density of the model. Based on a simple decomposition, the one step ahead predictive likelihood provides an alternative estimate of the marginal likelihood. The results from this exercise confirm the ordering of the models implied by the modified harmonic mean estimator. However the resulting marginal likelihood estimates appear to be less sensitive to the prior specification.

To the best of our knowledge, this is the first paper that explicitly compares the implications of the choice of time varying parameter vs. regime switching approaches for monetary policy dynamics. There is a growing literature that compares the forecasting performance of alternative macroeconomic models. Clark and Ravazzolo (2014) compare the forecasting performance of a variety of AR and VAR macro models using different specifications of changing volatility of the shocks. D'Agostino et al. (2013) use a tvp model with stochastic volatility and compare its forecasting performance with various other models. Both these papers focus on forecasting, use reduced form models and do not study any regime switching models. Perhaps the two papers that are closest to ours are Koop et al. (2009) and Barnett et al. (2014). Koop et al. (2009) use a mixture innovation model to examine the transmission mechanism of monetary policy. Their model allows for multiple structural breaks where the number of breaks is modeled using a hierarchical prior setup. Their model nests the tvp model but not the regime switching model. Finally, they allow for variation in the structural equations governing inflation and unemployment as well as the monetary policy equation. Using UK data, Barnett et al. (2014) compare a variety of models with time variation including regime switching models. However they use a different specification of regime switching, following the change point approach of Chib (1998), while in this paper we use the approach that is more common in macroeconomic analysis where transitions to past regimes are allowed. Additionally, their main focus is on forecasting and thus they use a reduced form model.

There is also a strand of the literature that tries to build a more flexible approach to modeling time variation, see for example Hamilton (2001), Koop and Potter (2007) and Giordani and Villani (2010). Koop and Potter (2010) consider a more general approach using the concepts of hypothetical data re-ordering and distance between observations. Their framework nests both regime-switching and time varying parameter approaches. Thus in principle there is no need for the researcher to choose between the tvp and rs approaches. But as mentioned earlier, in practice a large number of papers in applied macroeconomics choose one of the other approaches without trying the flexible approach.[4] This observation provides the fundamental motivation for our comparison

[4]Even though there is no empirical work as yet that uses these flexible models to study the dynamics of monetary

study.

The rest of the paper is organized as follows. The next section describes the model and explains the rs and tvp frameworks. Section 3 discusses the priors and gives an overview of the estimation methodology, while the estimation details are provided in an online appendix. The results are discussed in section 4 and section 5 provides some concluding remarks.

2 The Model

We consider a simple 3 variable VAR with p lags for $y_t = [u_t, \pi_t, i_t]'$ with unemployment (u_t), inflation (π_t) and the fed funds rate (i_t).

$$y_t = a_{0,t} + \sum_{j=1}^{p} b_{t,j} y_{t-j} + e_t, \quad e_t \sim N(0, \Omega_t) \tag{1}$$

$$\Omega_t = A_t^{-1} \Sigma_t \Sigma_t' A_t^{-1'} \tag{2}$$

Equation 2 represents the triangular decomposition of the reduced-form covariance matrix, where A_t is a lower triangular matrix with $1s$ on the diagonal, Σ_t is a diagonal matrix and $b_{t,j}$ is a 3 x 3 matrix of coefficients. This VAR is popular in the literature and has been used both in the regime switching case (see Sims et al. (2008)) and in the time varying parameter case (see Primiceri (2005)). We can write each equation of the VAR as

$$y_{nt} = z_{nt}' \beta_n + e_{nt} \tag{3}$$

for $n = 1, 2, 3$ and $z_{nt} = z_t = [1, y_{t-1}', ..., y_{t-p}']$ is 1 x k and β_n (which stacks the nth rows of the $b_{t,j}$ matrices) is k x 1, with $k = 3p + 1$. We can now stack the $z_{n,t}$ into a 3 x r matrix Z_t and β_n into a r x 1 vector ($r = 3k$) to get the following equation

$$y_t = Z_t \beta_t + A_t^{-1} \Sigma_t \varepsilon_t, \quad \varepsilon_t \sim N(0, I) \tag{4}$$

policy we think it is a promising area for future research.

with

$$Z_t = \begin{bmatrix} z_t' & 0 & 0 \\ 0 & z_t' & 0 \\ 0 & 0 & z_t' \end{bmatrix}, \beta_t = \begin{pmatrix} \beta_{u,t} \\ \beta_{\pi,t} \\ \beta_{i,t} \end{pmatrix} \tag{5}$$

To give this reduced-form VAR a structural interpretation some identification restrictions are required. The most common identification assumption in the monetary policy literature is the recursive (or triangular) identification, see Christiano et al. (1999) for a survey. In this identification scheme the fed funds rate is ordered last, with the implied assumption that unemployment and inflation do not react contemporaneously to changes in the fed funds rate but only with a lag. The fed funds rate is allowed to react contemporaneously to inflation and unemployment. With this assumption in place, the matrix governing the contemporaneous relationships between the three variables exactly coincides with the lower-triangular matrix A_t that comes from the triangular-decomposition of the reduced-form covariance matrix Ω_t. Alternative identification approaches have been used in the literature. See Gambetti et al. (2008) for an example of sign restrictions and Sims and Zha (2006) for non-recursive zero restrictions. Arguably, the recursive approach remains the most popular approach and this provides motivation for its use in the current paper.

Notice that the reduced form covariance matrix Ω_t is indexed by t. One component of the time dependency comes from time variation in A_t, which represents the time variation in the contemporaneous relationships between the three variables. Additionally the standard deviation of the structural shocks Σ_t is also allowed to depend on t. There is a large literature that has documented changes in the variance of the shocks, for example see Sims and Zha (2006) and Primiceri (2005). Thus we will allow for the variance of the shocks to all three variables to change over time. Since the focus of this paper is modeling monetary policy dynamics we will consider time variation in the parameters only in the monetary policy equation, the 3rd equation in the system shown above. This is a reasonable assumption as Sims and Zha (2006) find that the specification with changes only in the monetary policy equation fit the data better than a specification where parameters of the other equations change as well.[5] Thus in this paper we will consider models

[5]Additionally, Lhuissier and Zabelina (2015) provide some evidence that once you account for time variation in

where there are changes only in the monetary policy equation (referred to with a b), only in the variance of the shocks (v) and changes in both (vb). Thus our vb specification in the rs case is the same as the specification used in Sims et al. (2008), which they refer to as the "variance-with-policy-change" model. In the tvp case, the vb specification is identical to the model of Primiceri (2005) with the exception that only the coefficients of the monetary policy equation are allowed to change.

The parameters that are allowed to change over time have a t subscript. Note that with the structural interpretation the parameters $\alpha_{31,t}$ and $\alpha_{32,t}$ of the A_t matrix represent the contemporaneous response of interest rates to unemployment and inflation respectively. For expositional clarity the equation $y_t = Z_t\beta_t + A_t^{-1}\Sigma_t\varepsilon_t$ can be expanded as follows

$$\begin{pmatrix} u_t \\ \pi_t \\ i_t \end{pmatrix} = Z_t \begin{pmatrix} \beta_u \\ \beta_\pi \\ \beta_{i,t} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{u,t} & 0 & 0 \\ 0 & \sigma_{\pi,t} & 0 \\ 0 & 0 & \sigma_{i,t} \end{pmatrix} \varepsilon_t \tag{6}$$

2.1 Regime Switching

In the regime switching framework the parameters of the interest rate equation in $\beta_{i,t}$ and $A_{i,t} = [\alpha_{31,t}, \alpha_{32,t}]$ and all the parameters in Σ_t depend on an unobserved Markov switching variable that can take on integer values. In the baseline specification we model the coefficients of the monetary policy equation governed by the Markov switching variable $s_{1,t}$ which is independent from the variable $s_{2,t}$ that governs the switching of the variances. We can write the model as

$$\begin{pmatrix} u_t \\ \pi_t \\ i_t \end{pmatrix} = Z_t \begin{pmatrix} \beta_u \\ \beta_\pi \\ \beta_i(s_{1,t}) \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31}(s_{1,t}) & \alpha_{32}(s_{1,t}) & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_u(s_{2,t}) & 0 & 0 \\ 0 & \sigma_\pi(s_{2,t}) & 0 \\ 0 & 0 & \sigma_i(s_{2,t}) \end{pmatrix} \varepsilon_t \tag{7}$$

the variance of the shocks then time variation is not required in a key parameter in the inflation equation.

Both the Markov switching processes $s_{1,t}$ and $s_{2,t}$ are governed by transition matrices that are left unrestricted.[6] Note that we can combine the two processes $s_{1,t}$ with M_1 regimes and $s_{2,t}$ with M_2 regimes into the process s_t with $M = M_1$ x M_2 regimes. Thus the estimation involves estimating M different sets of parameters for $\beta_i(s_t)$,$A_i(s_t)$ and $\Sigma(s_t)$. The transition matrix contains the parameters $p(s_t = j|s_{t-1} = i) = p_{ij}$. We have also tried the specification where both the monetary policy coefficients and the variances depend on the same Markov process, but found that the data prefer the independent specification. This same finding is also reported in Sims and Zha (2006). In the baseline results we consider up to 4 total regimes.

2.2 Time varying parameter

In the time varying parameter framework the parameters are modeled as latent variables where a law of motion is specified for their dynamics. The most common approach in the monetary policy literature is to model the parameters as following random walks.

$$\begin{pmatrix} u_t \\ \pi_t \\ i_t \end{pmatrix} = Z_t \begin{pmatrix} \beta_u \\ \beta_\pi \\ \beta_{i,t} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ \alpha_{21} & 1 & 0 \\ \alpha_{31,t} & \alpha_{32,t} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{u,t} & 0 & 0 \\ 0 & \sigma_{\pi,t} & 0 \\ 0 & 0 & \sigma_{i,t} \end{pmatrix} \varepsilon_t \tag{8}$$

$$\beta_{i,t} = \beta_{i,t-1} + \nu_t, \qquad \nu_t \sim N(0, Q) \tag{9}$$

$$A_{i,t} = A_{i,t-1} + \zeta_t, \qquad \zeta_t \sim N(0, S) \tag{10}$$

$$log(\sigma_{j,t}) = log(\sigma_{j,t-1}) + \eta_{j,t}, \qquad \eta_t \sim N(0, W), j \in \{u, \pi, i\} \tag{11}$$

The covariance matrix W of the innovations to the log volatility process is assumed to be diagonal following Cogley and Sargent (2005). Thus the tvp setup allows for a break in the parameters in every time period, where the estimates of Q, S and W will govern the amount of estimated time variation.

[6] An alternative is the change point model, see for example Chib (1998). However, in the monetary policy literature Markov switching with unrestricted transition matrix is the more popular approach.

3 Priors and Estimation

3.1 Priors

For the rs case we use symmetric priors, so that regardless of the regime the prior distribution is the same. For the coefficients we use the so-called "Minnesota" style prior, which provides shrinkage towards a random walk.[7] Specifically the coefficients of the VAR are assumed to follow a normal prior $\beta \sim N(\underline{m_\beta}, \underline{M_\beta})$. Note that the β includes the parameters of both the monetary and non-monetary equations. $\underline{m_\beta}$ is set to 1 for the parameters corresponding to the first own lag and the rest are set to zero. The prior variance for coefficients of each equation is set in the following way. $\sqrt{\underline{M_\beta}} = \frac{\lambda_0 \lambda_1}{\delta_j L(j)^{\lambda_3}}$ where δ_j is the standard deviation of a univariate autoregression of equation j and $L(j)$ represents the order of the lag in equation j. This prior variance embodies the idea that own lags are more likely to be important predictors than lags of other variables and the predictive power diminishes as the lag length increases. Following Sims and Zha (1998) we set $\lambda_0 = 1$, $\lambda_1 = 0.2$ and $\lambda_2 = \lambda_3 = \lambda_4 = 1$. The parameters of the A matrix are assumed to have a normal prior with mean zero and a large variance. The inverse of the standard deviations σ_j^{-1} are assumed to have a Gamma prior, again with priors being symmetric across regimes. Finally for the transition matrix P which contains the regime probabilities $p_{i,j}$ we use a Beta prior when there are two regimes and a Dirichlet prior when there are more than two regimes. The prior parameter vector $\widetilde{\alpha}$ of the Beta and Dirichlet distributions are set so that they imply a probability of 0.85 of staying in the same regime and an equal probability of $\frac{1-0.85}{M-1}$ of moving to any another regime, where M is the total number of regimes. The priors are thus summarized as follows with $\underline{V_A} = 10{,}000$, $\underline{k} = \underline{\theta} = 1$ and $\underline{m_\beta}$, $\underline{M_\beta}$ and $\widetilde{\alpha}$ set as described above

$$A \sim N(0, \underline{V_A}) \tag{12}$$

$$\beta \sim N(\underline{m_\beta}, \underline{M_\beta}) \tag{13}$$

$$\sigma_j^{-1} \sim G(\underline{k}, \underline{\theta}) \tag{14}$$

$$P \sim Dir(\widetilde{\alpha}) \tag{15}$$

[7] As a robustness test, we have also used a training sample prior which gives similar results.

In the tvp case, to keep the priors similar to the rs case, we use a Minnesota-style prior for the constant parameters in β with the same prior parameters. For the priors regarding the latent variables, we follow Primiceri (2005) and use a training sample approach. The first 10 years of the sample is used to estimate a constant parameter VAR by OLS. Then the priors for the initial values of the latent variables are based on OLS estimates, $log(\hat{\sigma}_{OLS})$, $\hat{A}_{i,OLS}$ and $\beta_{i,OLS}$. The variance of the shocks in the random walk processes are assumed to follow Inverse-Wishart distributions with the scale matrix that depends on the constant parameters, k_Q, k_W and k_S. For the benchmark results we use the values $k_Q = 0.01$, $k_S = 0.1$ and $k_W = 0.01$ that are very common in literature, see Primiceri (2005), D'Agostino et al. (2013) and Cogley and Sargent (2005) among others. Thus the prior setup is summarized as follows

$$\beta_{u\pi} \sim N(\underline{m_{\beta_{u\pi}}}, \underline{M_{\beta_{u\pi}}}) \quad (16)$$

$$\alpha_{21} \sim N(0, 10, 000) \quad (17)$$

$$log(\hat{\sigma}_0) \sim N(log(\hat{\sigma}_{OLS}), I) \quad (18)$$

$$\beta_{0,i} \sim N(\hat{\beta}_{i,OLS}, 4.V(\hat{\beta}_{i,OLS}) \quad (19)$$

$$A_{0,i} \sim N(\hat{A}_{i,OLS}, 4.V(\hat{A}_{i,OLS})) \quad (20)$$

$$Q \sim IW(k_Q^2.40.V(\hat{\beta}_{i,OLS}), 40) \quad (21)$$

$$W \sim IW(k_W^2.4.I_n, 4) \quad (22)$$

$$S \sim IW(k_S^2.3.V(\hat{A}_{i,OLS}), 3) \quad (23)$$

3.2 Estimation

For the tvp specification we use the algorithm outlined in Primiceri (2005). This is a Gibbs Sampler which uses the simulation smoother of Carter and Kohn (1994) and a mixture of normal approximation for the stochastic volatility based on Kim et al. (1998). We make sure to use the correct ordering as pointed out by Del Negro and Primiceri (2013). For the rs case we use a single block random-walk Metropolis-Hastings (M-H) algorithm. Although a Gibbs Sampler can also be constructed for the models used here, we found that the Metropolis-Hastings algorithm had good convergence properties and was much more convenient to use.[8] All the estimation details are provided in an online appendix.

4 Results

The estimation uses quarterly data as follows. The unemployment rate is for civilians 16 years and over. Inflation is the annualized percentage change in the GDP Deflator. The interest rate is the fed funds rate at an annualized rate. The data sample runs from 1954:Q4 to 2008:Q3. We use the first 10 years as a training sample so the effective sample runs from 1964:Q3 to 2008:Q3. The post-financial crisis sample is not used due to the zero lower bound constraint on the fed funds rate. The model is estimated using two lags.

4.1 Marginal Likelihood

With a Bayesian estimation framework, a natural way to perform model comparison is to calculate the posterior odds. With a priori equal weight associated to each model the Bayesian posterior odds ratio boils down to comparing the marginal likelihood. Let $y^{1:t} = [y_1, y_2, ...y_t]$ denote a vector of data from period 1 up to t. Next, let θ denote all the constant parameters and ξ^T the latent variables. Gathering all the parameters and latent variables in $\Theta = \left\{\theta, \xi^T\right\}$, the marginal likelihood is given by

$$p\left(y^{1:T}\right) = \int p(y^{1:T}|\Theta)\pi(\Theta)d\Theta \tag{24}$$

We use the modified harmonic mean estimator that is common in the literature to calculate the marginal likelihood, as detailed in appendix section A-1.

Table 1 shows the log marginal likelihood for the different specifications of the models. The top row shows that for the constant parameter VAR the log marginal likelihood is -484.85. This

[8] The basic random-walk M-H algorithm is the same for the various rs specifications and requires just changing the log-likelihood file while a Gibbs Sampler would have to be specifically tailored for each different specification.

is useful as a benchmark to compare with the other specifications. The highest marginal likelihood is achieved for the regime switching variance only model with 3 regimes (rs v3) with a value of -393.42. Looking only at the tvp case, the best fit model is also the one with only variation in the shock variances with a log marginal likelihood of -396.76. Thus the marginal likelihood suggests that data prefer change in shock variances rather than a setting that has changes in the monetary policy rule, or changes in both the variances and the policy rule. Overall our results are consistent with the literature that supports what has come to be known as the "good luck" hypothesis, see Primiceri (2005), Liu et al. (2011) and Sims and Zha (2006). Specifically, this literature concludes that the data prefer a specification with only changes in the variance of the shocks as opposed to changes in policy parameters (or both changing). Moreover, our results confirm that this conclusion does not depend on whether one uses a rs or tvp framework.

The marginal likelihood for each of the rs specifications is higher than the constant parameter VAR. But for the tvp b and tvp vb specifications, the marginal likelihood is actually lower than the constant parameter case. For the TVP specification the log marginal likelihood of the tvp b specification is -821.35, which is significantly lower than the value for the tvp v model or the rs b model with 2 or 3 regimes. What is the reason for such a low value? To better understand this, the next two columns in the table show the value of the log likelihood and the log prior evaluated at the posterior mean of the parameters. The analytical form of the marginal likelihood is not known for these models and thus we cannot exactly formulate the contribution of the prior and likelihood. Nonetheless, this exercise can provide some useful insight. Intuitively, the likelihood is a measure of the fit, while the prior contributes to a penalty for over-parametrization, with the penalty getting higher as the posterior has lesser overlap with the prior. The value of the log prior for the baseline case, for all the rs specifications and for the tvp v case are between -30 and -63. On the other hand, for the tvp b and tvp vb specification the log prior at the posterior mean is -375.84 and -402.99 respectively. Thus the prior seems to be lowering the marginal likelihood dramatically for the tvp b and tvp vb cases. We can further analyze the contribution of the various parameter blocks to the log prior, as show in table 2. This table shows that the biggest contribution to the log prior comes from Q, the covariance matrix of the shocks to the time varying coefficients of the monetary policy rule. One concern is that large negative values reflect a situation where the posterior distribution of Q has very little overlap with the prior distribution. To investigate this, we consider the following exercise. Q has an Inverse-Wishart prior, $Q \sim IW(\underline{\nu}, \underline{Q})$ with shape parameter $\underline{\nu} = 40$ and scale matrix $\underline{Q} = k_Q^2.40.V(\beta_{i,OLS})$ which depends on the variance of coefficients from a training sample regression, as explained in section 3.1. We can evaluate the log density of this prior distribution at the mode which is given by $\underline{Q}(\underline{\nu} - 7 - 1)$. The log prior density evaluated at the mode is -327.99, which is not much higher than the values -375.84 and -402.99 reported in table 1. In other words, the posterior distribution of Q has reasonable overlap with the specified prior distribution. Taking this idea a step further we can consider a hypothetical situation where the prior distribution is the same as the estimated posterior. Even in this case the log prior density at the mode is not high enough to change our conclusion. This suggests that by construction the Inverse-Wishart prior adds a relatively large penalty compared to the other parameter blocks. Since the Inverse-Wishart distribution is very common in the time varying parameter literature, this is an important consideration for any studies that use the marginal likelihood for model comparison. There are papers that have hinted at the sensitivity of the marginal likelihood to priors for tvp models without showing the contribution of the priors, see Campolieti et al. (2014) and Koop et al. (2009) among others.[9] One option is to use simple alternative measures like the Bayesian Information Criterion or the expected Log Likelihood. Another option is to use the predictive density, as we discuss in the next subsection.

To shed a little more light on the fit of the different models, figure 1 plots the conditional log density of the tth observation, evaluated at the posterior mean of the estimates. The axes in each row of the figure are set to the same values for ease of comparison. A common theme is that the conditional log density value is low for all specifications around 1980 with the lowest value reached in 1980:Q4. This coincides with the reserves targeting experiment in the early years of Paul Volcker's regime. For the tvp models there is also a dip in the conditional log density around 1975 (especially for the tvp v model) while the regime switching models tend to fit well during this

[9] Additionally, Chan and Grant (2015) show in a simulation study that the marginal likelihood estimates can have a significant bias but their study uses a simpler unobserved components model.

period. As will be clear from figures and discussion below, this is because the regime switching models are good at capturing big abrupt changes while the tvp model is designed to better capture gradual movements.

4.2 Forecast Performance and Predictive Likelihood

In this section we take advantage of the Bayesian estimation framework and analyze the posterior predictive density. There are two main reasons for this. First, this helps in comparing the forecasting performance of the different models. Second, we can get an alternative estimate of the marginal likelihood by exploiting a decomposition that relates the marginal likelihood to the one step ahead predictive likelihoods. These exercises require the use of a recursive estimation scheme as performed in Clark and Ravazzolo (2014) and D'Agostino et al. (2013). For the first run we estimate the model using data from 1965:Q3 to 1975:Q3. Then we add 1 quarter of data and re-estimate the model using data from 1965:Q3 to 1975:Q4, and so on till the last run where the full sample is used.

First, using these estimates we forecast up to 8 quarters ahead. The forecast performance is evaluated using the root mean square forecast error (rmsfe). The numbers are reported in table 3. We will focus on the results for the fed funds rate but results for unemployment and inflation are provided as well. First comparing across the rs and tvp specifications, we notice that the rmsfe (for all horizons) are lower for rs models with changing variances compared to the corresponding tvp specifications. This suggests that in addition to providing a better in sample fit, the rs models with changing variances also perform well out of sample. Within the rs specifications, the rs v3 model provides the best forecasts and allowing for time variation in the monetary policy parameters increases the rmsfe. This is consistent with the picture emerging from the marginal likelihood calculations. Now turning to comparison within tvp models, we notice that the forecast performance aligns with the marginal likelihood comparison as well. But here the values of the rmsfe for the three different specifications are closer to each other. One way to highlight this is to notice that the rmsfe for the tvp b model is actually lower than that for the rs b2 model, whereas the marginal likelihood suggested a better fit for the rs b2 model. Overall, the forecasting exercise gives results that are consistent with the marginal likelihood calculations using the modified harmonic mean estimator.

Next we consider the predictive likelihood. Appendix section A-2 shows the details of how the predictive likelihood is evaluated. The evaluation of the one step ahead predictive likelihood ($PL(t)$) provides an alternative way to calculate the marginal likelihood. It can be shown that (for example see Geweke and Amisano (2010))

$$\log p(y^{t_1:T}) = \sum_{t=t_1}^{T} \log PL(t) \tag{25}$$

Thus the log marginal likelihood for estimation using data from t_1 (1975:Q3) to T (2008:Q3) is the sum of the one step ahead log predictive likelihoods from the recursive forecasting exercise. Note that this number is not directly comparable to the marginal likelihood numbers reported in table 1, where the full sample 1965:Q3 to 2008:Q3 is used. Nevertheless, given the high overlap with the full sample, the sum above should provide a good indicator of the fit of the model. This sum of the log predictive likelihoods is reported in table 4. Comparing within the rs and tvp framework, we conclude that the variance only models have the best fit, consistent with the marginal likelihood conclusions obtained from table 1. Moreover, the ranking of the three different specifications for the rs models is the same as that implied by table 1(rs v fits best, followed by rs vb and then rs b2). However for the tvp models, the order of tvp b and tvp vb is switched. Interestingly, we do not observe the big deterioration of the marginal likelihood for the tvp b and tvp vb cases that was observed with the modified harmonic mean estimator. This suggests that the "penalty" imposed by the Inverse-Wishart prior specification on the marginal likelihood seems to be particularly affecting the modified harmonic mean estimator and does not negatively affect the predictive density of the model as much.

An added advantage of the predictive likelihood decomposition is that it can shed light on how individual data observations contribute to the evidence for each model. Figure 2 shows the one-step ahead log predictive likelihood for each time period. The y-axes are aligned to the same values for each row in the figure. As was seen in figure 1, it is clear that predictive likelihood is the lowest around the end of 1980. What is striking is the magnitude of the contribution of one single data point in lowering the evidence for the tvp model: 1980:Q4. In other words, once you ignore the 1980:Q4 observation, then the predictive likelihood evidence for the rs and tvp models is very close indeed. As will be discussed in the next subsection, the rs models display quick switches between regimes around the end of 1980 (see figures 3 and 6). On the other hand, the tvp models cannot account for large discrete changes (see figures 4 and 6). This fact explains the superior performance of the rs models around the end of 1980.

In terms of forecasting performance for the tvp models, our results are consistent with Clark and Ravazzolo (2014), who find that the VAR with stochastic volatility in the shocks outperforms a VAR where both the parameters and variance of shocks are allowed to change. Additionally, Giordani and Villani (2010) find that models that allow large infrequent shifts in the variance of the shocks outperform gradual continuous movements. This is also consistent with our results from tables 1,3 and 4 and figure 2. Similar to our paper, these two studies use US data. On the other hand, for UK data, Barnett et al. (2014) find that the tvp models outperform rs models in terms of forecasting. Although their specification of regime switching uses the change point framework. This specification puts restrictions on the transition matrix of the regime indicator, which rules out transitions to past regimes.

Based on the results in this section, we can make some simple recommendations for applied researchers who care about the fit of the model and forecasting performance. Models with changes in shock variance are the best fit models for both rs and tvp specifications. Moreover, if the researcher does not have any a priori reason for choosing rs or tvp models, then we recommend using the rs specification. For researchers that insist on using a specification with changes in the monetary policy rule, the results still suggest using a rs approach from a fit and forecasting perspective.[10] Next, we analyze whether the choice of tvp vs. rs has important implications for structural questions of interest. The typical strategy in the empirical literature is to consider either the rs or the tvp framework and conduct analysis of economic issues within that particular

[10]See subsection 4.4 for monetary policy implications of choosing rs vs tvp within this class of models.

framework. Armed with the same underlying VAR model, identifying assumptions, data sample and priors (as much as possible) we can now evaluate whether researchers conducting analysis in only the rs framework or tvp framework would arrive at the same answers to important questions of interest.

4.3 Change in Shock Variances

The best fit model in both the rs and tvp framework is the variance only model and we compare the variance dynamics implied by each model. Figure 3 shows the smoothed probabilities of the variance regimes in the rs v3 model, with the figures arranged in ascending order of variance starting with the lowest variance in the top left. We see a pattern that is commonly found in the literature. The lowest variance regime is in place for the Great Moderation period from mid-1980s to around 2008 and also in the late 1960s. The 1970s and 1980s are characterized by switches between the medium and high variance regimes. The high variance regime is prevalent for two episodes, one in the mid-1970s and one in the early 1980s. A similar picture emerges from the tvp v model. Figure 4 plots the posterior mean of the time varying standard deviation $\sigma_{j,t}$ from the tvp v model. The standard deviation of the shocks in all three equations display a higher level in the 1970s and early 1980s with a significant decline around the mid 1980s. Moreover the two peaks in the standard deviation of the fed funds rate equation (in the late 1970s and early 1980s) correspond to the third volatility regime being in place in the rs v3 model. As one would expect, the regime switching approach suggests more abrupt and frequent changes in the volatilities in the 1970s. But overall, we can conclude that the two best fit models paint a similar picture about the dynamics of shock variances.

Next, we consider an important question that is the focus of a large number of monetary VAR studies: What is the effect of monetary policy on the economy? To get around the endogeneity issue the literature has focused on identifying monetary policy shocks. In the structural VAR framework with the recursive identification, these are the disturbances in the interest rate equation that can be backed out using the estimates of the covariance matrix of the reduced-form VAR. We analyze whether using a regime switching approach or a time varying parameter approach gives any discernible differences in the identified monetary policy shocks. Figure 5 plots the monetary policy shocks from both the rs v3 model and tvp v model. Unsurprisingly, it is clear from the figure that the two measures of monetary policy shocks are almost identical. The estimates of the coefficients are also very similar in the two specifications.[11] Thus an important conclusion is both methods deliver almost identical monetary policy shocks and imply the same pattern for the change in the variance of shocks.

4.4 Change in Monetary Policy Rule

Next we turn our attention to comparing models that just allow changes in the monetary policy rule. Even though the marginal likelihood for these models is lower than the variance only models, this exercise is still of interest. There seems to be a view among economists that monetary policy has changed since the 1970s, see Clarida et al. (2000) and the large related literature.[12] The following exercise aims to shed light on whether the decision of rs vs tvp has important implications for characterizing changes in monetary policy.

We first consider the long run response to inflation in the tvp model. There are two factors that can contribute to a change in the long run response. First, the response of the fed funds rate to current and lagged inflation can change. Second, there can be a change in how the fed funds rate responds to lagged fed funds rate (or so-called "inertia"). Panel (a) in figure 6 shows the long run response to inflation from the tvp b model. This long run response in the tvp model rises gradually from the mid 1960s to mid 1970s, when there is a small dip. The peak of the response is in the early years of the Volcker regime and since the mid 1980s the response has been relatively stable. Interestingly, the Taylor principle is not satisfied only in the first four quarters of the sample (1965:Q1 - 1965:Q4).[13] The results from the rs b3 model paint a slightly different picture. Panels

[11] The parameter estimates are not show here, but are available upon request.

[12] See, Benati and Surico (2009) for additional motivation for considering a framework with just change in monetary policy coefficients.

[13] Note that the Taylor principle requires that interest rates increase more than one for one in response to an increase inflation.

(b), (c) and (d) show the smoothed probabilities of the three regimes. The color and line-style of the probabilities in these panels corresponds to that in panel (a) where the horizontal line shows the long run responses under each of the three regimes. The average of the long run response to inflation under the two models is similar but the dynamics are not. The regime probabilities show that the first regime (black line in (a) with probabilities in (b)) is in place for majority of the sample. The long run response to inflation in this regime is around 3, while the response is significantly lower in the other two regimes. The regime probabilities show that there are multiple switches between the first "hawkish" regime and the other two "dovish" regimes. Importantly, this suggests that there were multiple instances in the 1970s where the Taylor principle was not satisfied. This is in contrast with the tvp b model where this occurs only briefly in the beginning of the sample.

Another way to highlight this difference is by looking at the impulse response of the fed funds rate to an inflation shock. Figure 7 shows the response of the fed funds rate to a negative 1 unit inflation shock, which in this case can be interpreted as a supply shock. This response is shown in the top panel for parameter estimates for the time period 1982:Q3. The rs b3 model implies that the response was a lot more muted and short-lived relative to the tvp model. Formally, we compute the difference between these responses and also the 16th and 84th percentile credibility bands. These bands do not include zero and thus provide evidence for statistical significance in the difference.[14] This difference in the estimated dynamics of the policy coefficients has important implications for the identification of monetary policy shocks. Figure 8 plots the monetary policy shocks from the tvp b and rs b3 models. Unsurprisingly, there are big differences between the two series during the 1970s and early 1980s, where the tvp b specification finds slow gradual movements in the response to inflation while the rs b3 specification finds frequent jumps between the three regimes. One may expect that if the number of regimes in the rs b model are allowed to increase that the dynamic pattern may look similar to the tvp b case. But in practice, for this three variable model it is difficult to achieve identification for a specification with more than 3 or 4 regimes.[15]

[14]Of course, this date is chosen specifically to make this point and the differences are smaller on average.

[15]Sims et al. (2008) are able to achieve identification of more than 3 regimes in a 3 variable model but they use a restrictive form of time variation.

Thus we conclude it is likely that researchers using the rs or tvp model with changes allowed only in the monetary policy rule would not arrive at identical conclusions about changes in the behavior of monetary policy.

5 Conclusion

Regime switching and time varying parameter models are popular in applied macroeconomic research. In this paper we compare the performance of these models in evaluating structural changes in monetary policy. In a structural VAR setting we use Bayesian Markov-Chain Monte Carlo techniques to estimate the two specifications of time variation. We use both conventional marginal likelihood calculations and predictive density comparisons to evaluate the models. We find that the conventional marginal likelihood calculation in the time varying parameter case is somewhat sensitive to the prior specification. However, overall the marginal likelihood and predictive density comparisons paint a similar picture about the fit of the models. In both the rs and tvp cases, the best fit model is one where the variance of the shocks changes but not the parameters of the monetary policy rule. Thus researchers using either the rs or tvp framework would arrive at similar answers to monetary policy questions as long as they use the best-fit model. On the other hand, if researchers insist on using a specification where only the monetary policy parameters change, they would find potentially conflicting results about the changes in monetary policy.

Finally, our results highlight the sensitivity of marginal likelihood calculations to the prior specification. This points to the need of simulation studies to evaluate the robustness of the model comparison approaches in the time-varying parameter and regime switching approaches.

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Appendix

A-1 Modified Harmonic Mean Estimator

Let $y^{1:t} = [y_1, y_2, ...y_t]$ denote vector of data from period 1 up to t. Next let θ denote all the parameters and ξ^T the latent variables. Gathering all the parameters and latent variables in $\Theta = \left\{\theta, \xi^T\right\}$ the marginal likelihood is written as

$$p\left(y^{1:T}\right) = \int p(y^{1:T}|\Theta)\pi(\Theta)d\Theta \tag{A-1}$$

where $p(y^{1:T}|\Theta)$ is likelihood and $\pi(\Theta)$ is the prior. We will use the modified harmonic mean estimator of Gelfand and Dey (1994), with the truncated normal weighting function $f()$ suggested by Geweke (1999).[16]

$$p\left(y^{1:T}\right)^{-1} = \left[\frac{1}{D}\sum_{i=1}^{D}\frac{f(\Theta^{(i)})}{p(Y|\Theta^{(i)})\pi(\Theta^{(i)})}\right] \tag{A-2}$$

$\Theta^{(i)}$ represents the *ith* draw from the posterior distribution, with D representing the total number of draws. Given the high dimension of latent variables we make some simplifying assumptions to aid in of the computation of the marginal likelihood, following Justiniano and Primiceri (2008). First we assume an independent structure for both the priors and the weighting function for the parameters and the latent variables, $\pi(\theta, \xi^T) = \pi(\theta)\pi(\xi^T)$ and $f(\theta, \xi^T) = f(\theta)f(\xi^T)$. Next we assume that the weighting function of the latent variable is just equal to the prior, $f(\xi^T) = \pi(\xi^T)$. Given these assumptions, the marginal likelihood can be calculated using the simpler equation

$$p\left(y^{1:T}\right)^{-1} = \left[\frac{1}{D}\sum_{i=1}^{D}\frac{f(\theta^{(i)})}{p(Y|\theta^{(i)})\pi(\theta^{(i)})}\right] \tag{A-3}$$

[16] An alternative approach is to use the method of Chib (1995) for the Gibbs Sampler and Chib and Jeliazkov (2001) for the Metropolis-Hastings. In practice, we found that the Chib (1995) method required a large number of draws to converge and thus decided to use the modified harmonic mean method.

A-2 Predictive Likelihood

In this section we show the details involved in calculating the one-step ahead predictive likelihood, $p(y_{t+1}|y^{1:t})$. Recall that θ represent the constant parameters of the model, with $\xi^{1:t}$ being the time dependent latent variables and that $y^{1:t} = [y_1, y_2, ...y_t]$ denotes vector of data from period 1 up to t. The one-step ahead predictive likelihood is given by

$$p(y_{t+1}|y^{1:t}) \quad = \quad \int\int p(y_{t+1}, \theta, \xi^{1:t+1}|y^t) d\xi^{1:t+1} d\theta \qquad \text{(A-4)}$$

where the joint distribution can be factored as following

$$\begin{aligned} p(y_{t+1}, \theta, \xi^{1:t+1}|y^{1:t}) &= p(y_{t+1}|\theta, \xi^{1:t+1}, y^{1:t}) p(\theta, \xi^{1:t+1}|y^{1:t}) && \text{(A-5)} \\ &= p(y_{t+1}|\theta, \xi^{1:t+1}, y^t) p(\theta, \xi_{t+1}, \xi^{1:t}|y^{1:t}) && \text{(A-6)} \\ &= p(y_{t+1}|\theta, \xi^{1:t+1}, y^t) p(\xi_{t+1}|\theta, \xi^t, y^t) p(\theta, \xi^t|y^t) && \text{(A-7)} \end{aligned}$$

The last term in equation A-7 are represented by draws from posterior distribution obtained from the recursive estimation exercise using data up to time $t \in t_1, t_2, ...T$. t_1 represents 1975:Q3 while the last period in the sample T is 2008:Q3. For the tvp models, the draws from the middle term are obtained using the algorithm outlined in Cogley et al. (2005) and D'Agostino et al. (2013). Specifically, we first draw from normally distributed residuals of the time-varying parameters and then obtain draws of the time-varying parameters themselves using the random walk specification. For the rs models, we first obtain a forecast of the probabilities at $t+1$ using the estimated filtered probabilities and the transition matrix. Then we draw a uniform random variable to select a draw for the regime at $t+1$. Using the simulated ξ^{t+1} and θ from the posterior simulator and taking advantage of the Markov structure of the model, we can calculate the predictive likelihood using

$$PL(t+1) = \frac{1}{D} \sum_{i=1}^{D} p(y_{t+1}|\theta^{(i)}, \xi^{t+1(i)}, y^{1:t}) \qquad \text{(A-8)}$$

Model	Marginal Likelihood	Log Likelihood	Log Prior
Baseline	-484.85	-458.00	-33.48
	Regime Switching		
rs v2	-410.46	-366.46	-55.50
rs v3	-393.42	-354.72	-57.38
rs b2	-435.10	-404.53	-46.57
rs b3	-404.15	-378.04	-53.74
rs v2b2	-406.68	-368.44	-62.01
	Time Varying Parameter		
tvp v	-396.76	-302.29	-58.41
tvp b	-821.35	-448.12	-375.84
tvp vb	-837.36	-284.64	-402.99

Table 1: The table shows the marginal likelihood estimates using the modified harmonic mean estimator. The log-likelihood and log prior are evaluated at the posterior mean of the parameter estimates

	$\beta_{u\pi}$	Σ	α_{21}	S	Q	$\beta_{0,i}$	$A_{0,i}$	W	$\log(\sigma_0)$
tvp b	1.6	-17.84	-5.52	-30.76	-331.91	5.42	3.18	N/A	N/A
tvp vb	1.32	N/A	-5.52	-31.76	-338.05	5.42	3.18	-34.81	-2.76

Table 2: The table shows the contribution of the various parameter blocks to the value of the log prior evaluated at the posterior mean of the parameter estimates.

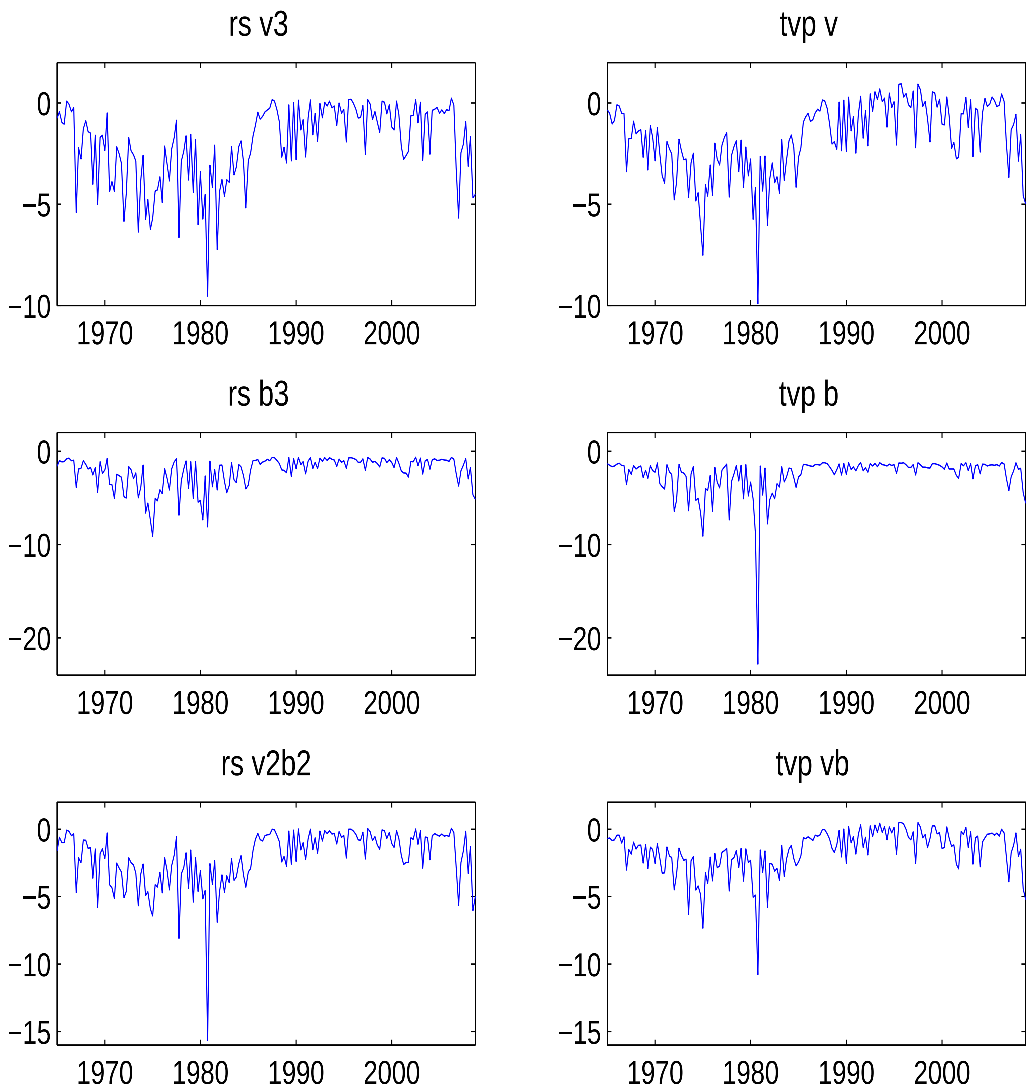
Horizon	rs v3	rs b2	rs v2b2	tvp v	tvp b	tvp vb
Federal Funds Rate						
1	0.94	1.06	0.93	0.95	0.98	0.99
2	1.45	1.68	1.44	1.50	1.59	1.63
4	2.09	2.38	2.08	2.16	2.25	2.43
8	3.17	3.50	3.29	3.26	3.31	3.78
Inflation						
1	1.00	1.03	1.03	1.02	1.02	1.01
2	1.20	1.25	1.25	1.23	1.24	1.23
4	1.56	1.65	1.66	1.57	1.61	1.58
8	2.24	2.49	2.55	2.33	2.45	2.48
Unemployment						
1	0.26	0.27	0.26	0.26	0.27	0.26
2	0.52	0.54	0.51	0.53	0.54	0.53
4	0.95	1.00	0.96	0.98	1.01	0.99
8	1.39	1.39	1.42	1.43	1.45	1.45

Table 3: This table shows the root mean squared forecast error calculated using the posterior mean for the different specifications.

Table 4: Sum of one step ahead log predictive likelihood

	Regime Switching
rs v3	-298.81
rs b2	-352.91
rs v2b2	-308.37
	Time Varying Parameter
tvp v	-304.90
tvp b	-380.00
tvp vb	-323.11

Figure 1: Log Likelihood evaluated at posterior mean



This figure plots the conditional log density of the tth observation, evaluated at the posterior mean of the estimates.

Figure 2: One step ahead log predictive likelihood

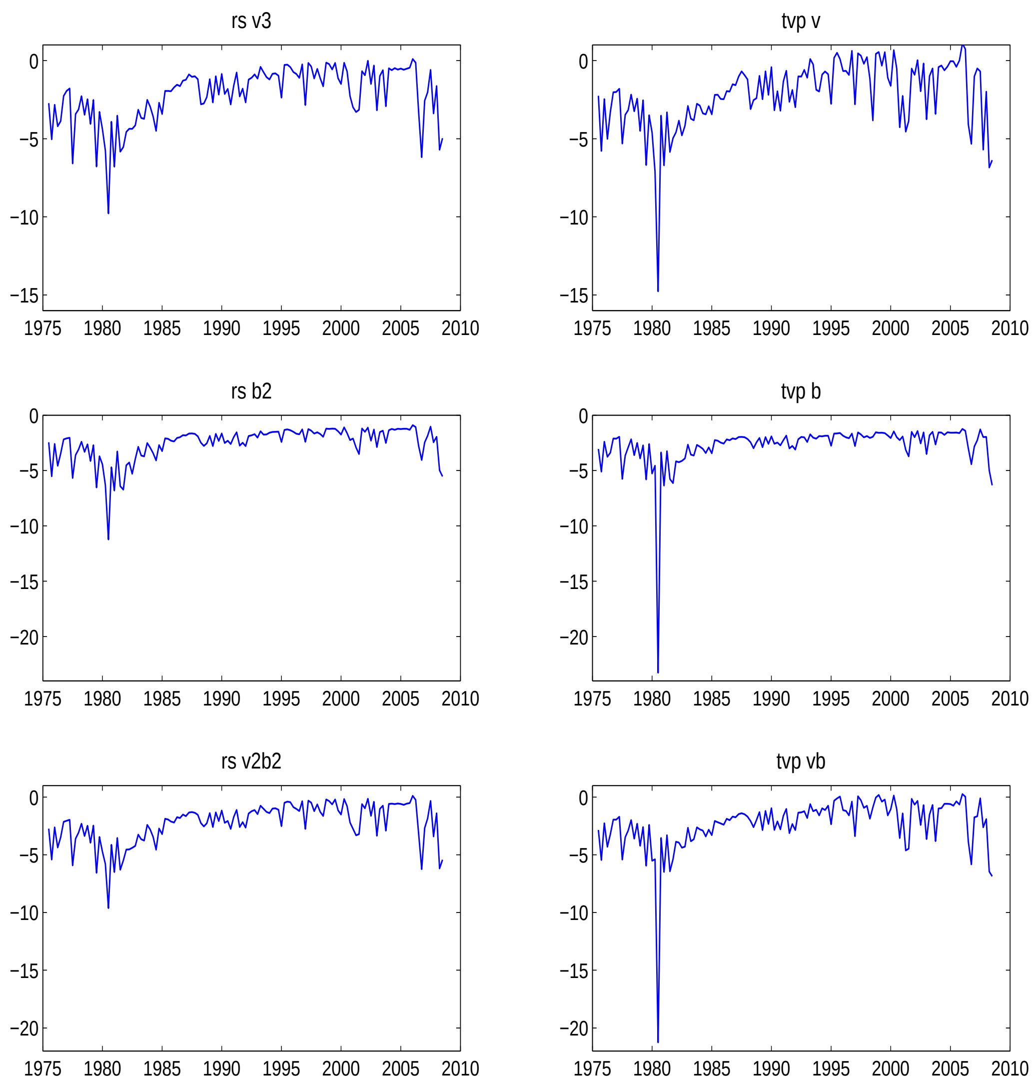
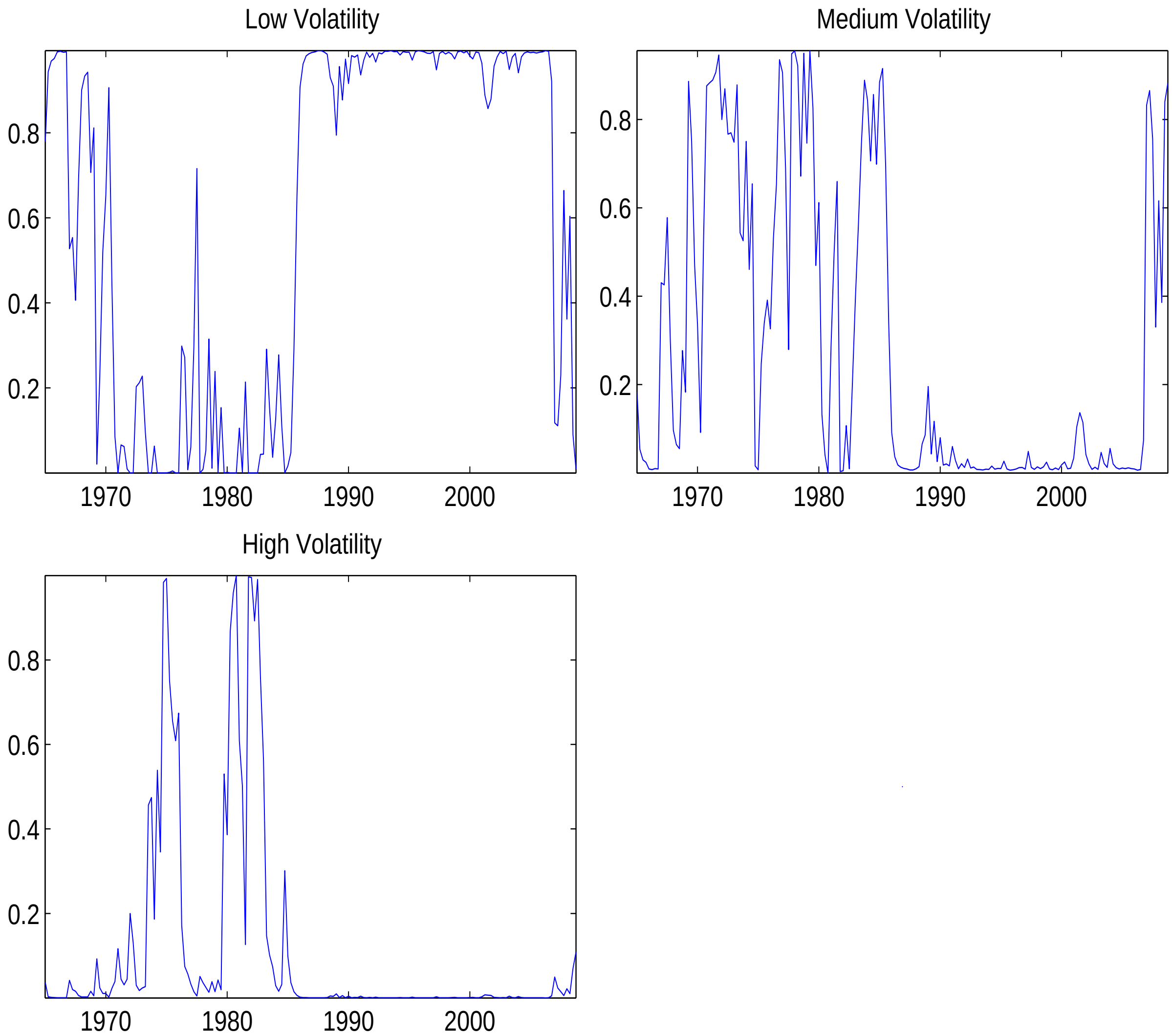
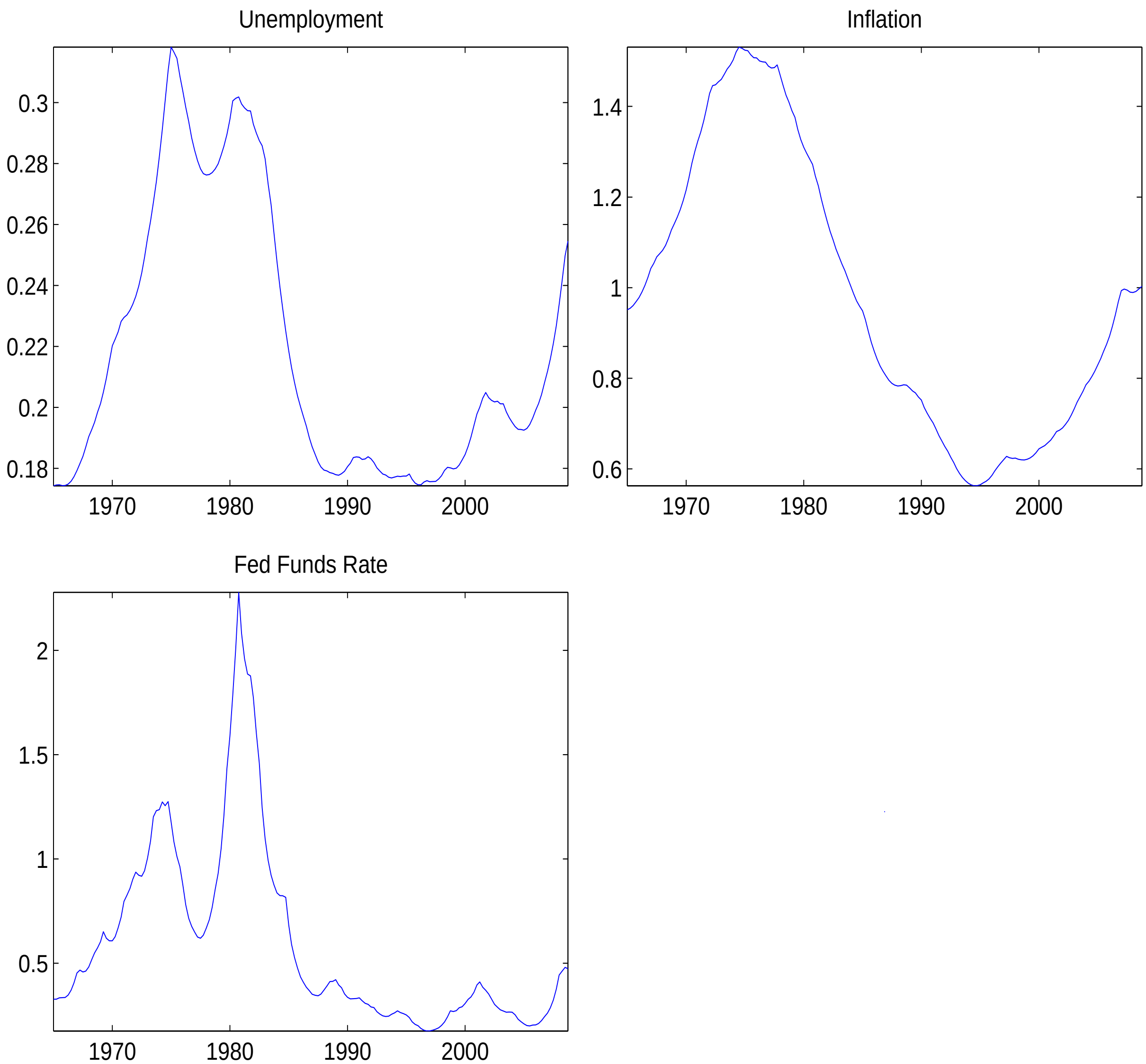


Figure 3: Smoothed probability of variance regimes



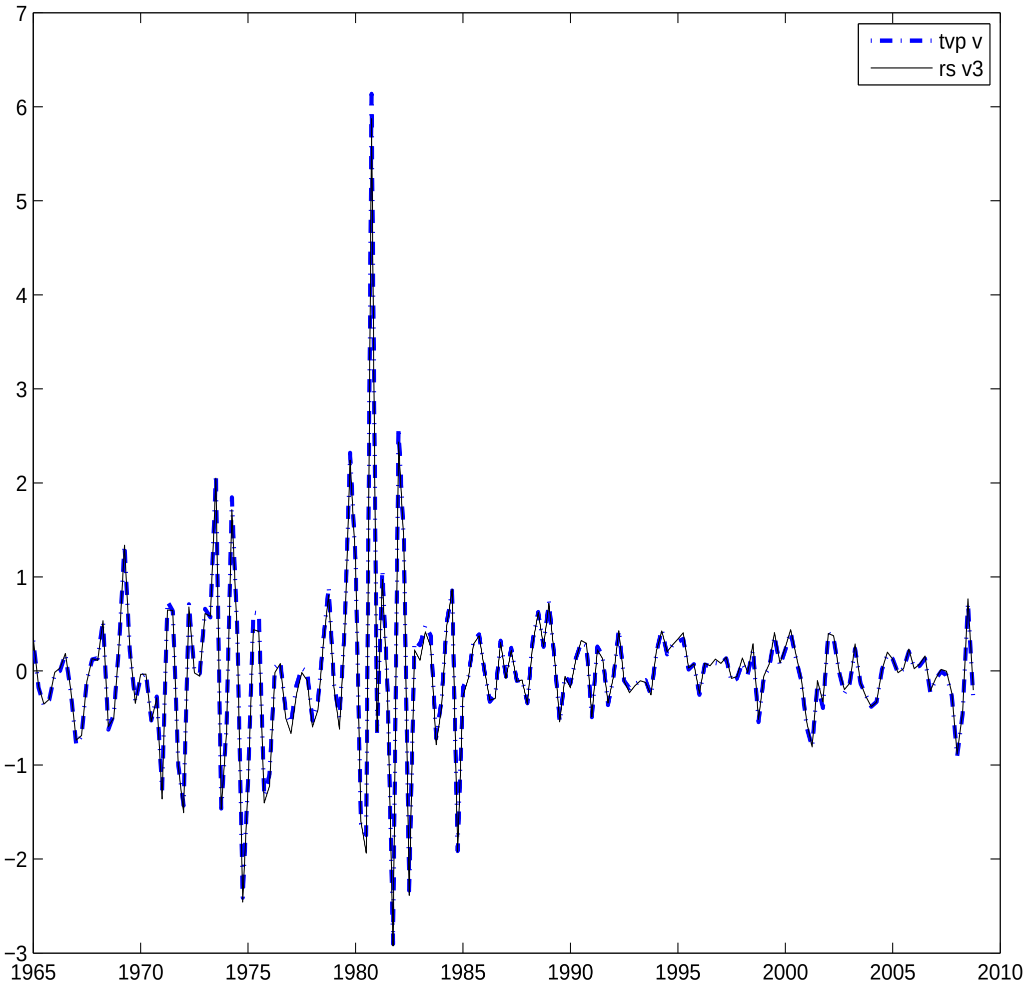
The figure shows the smoothed probability of the variance regimes at the posterior mean from the rs v3 model.

Figure 4: Posterior Mean of Standard Deviations



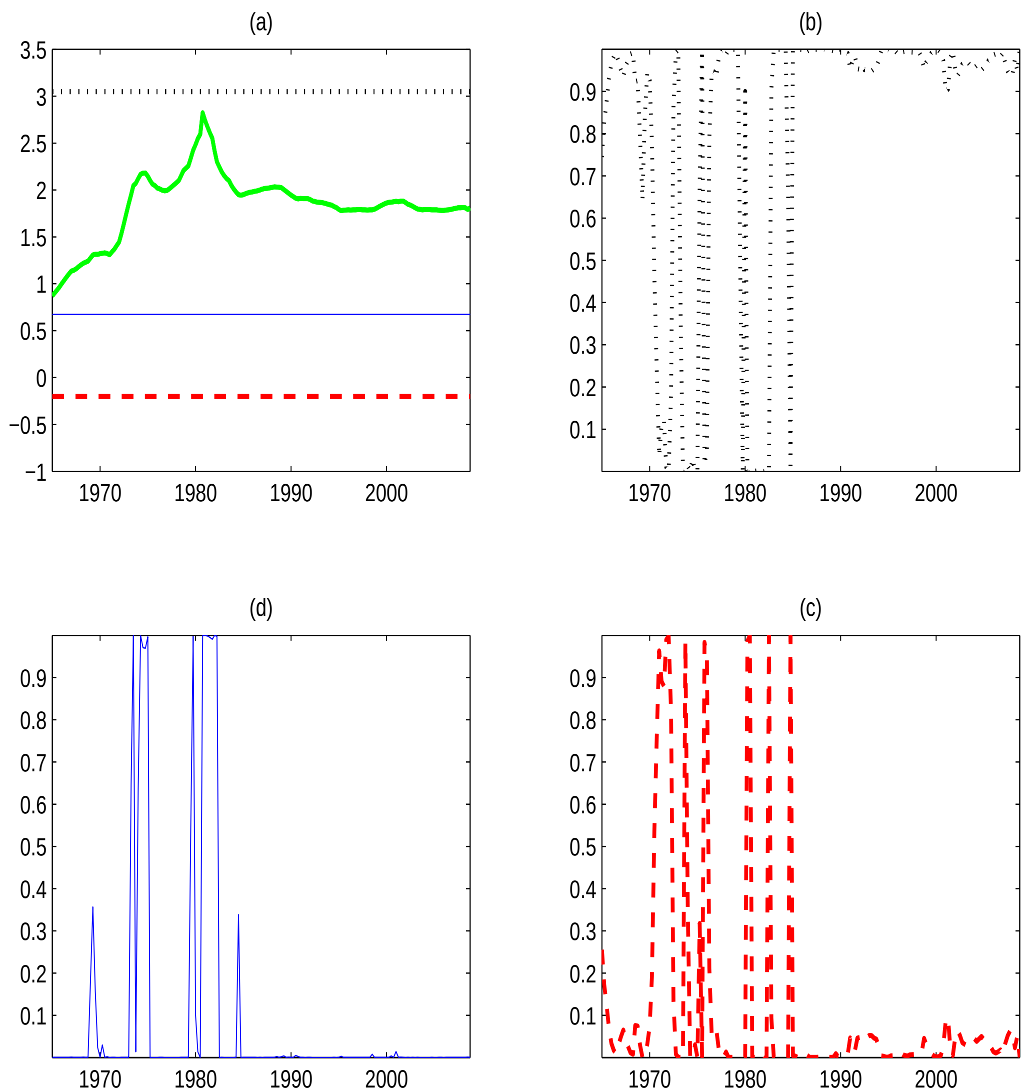
The figure shows the posterior mean of the time-varying standard deviations in the tvp v model.

Figure 5: Monetary Policy Shocks



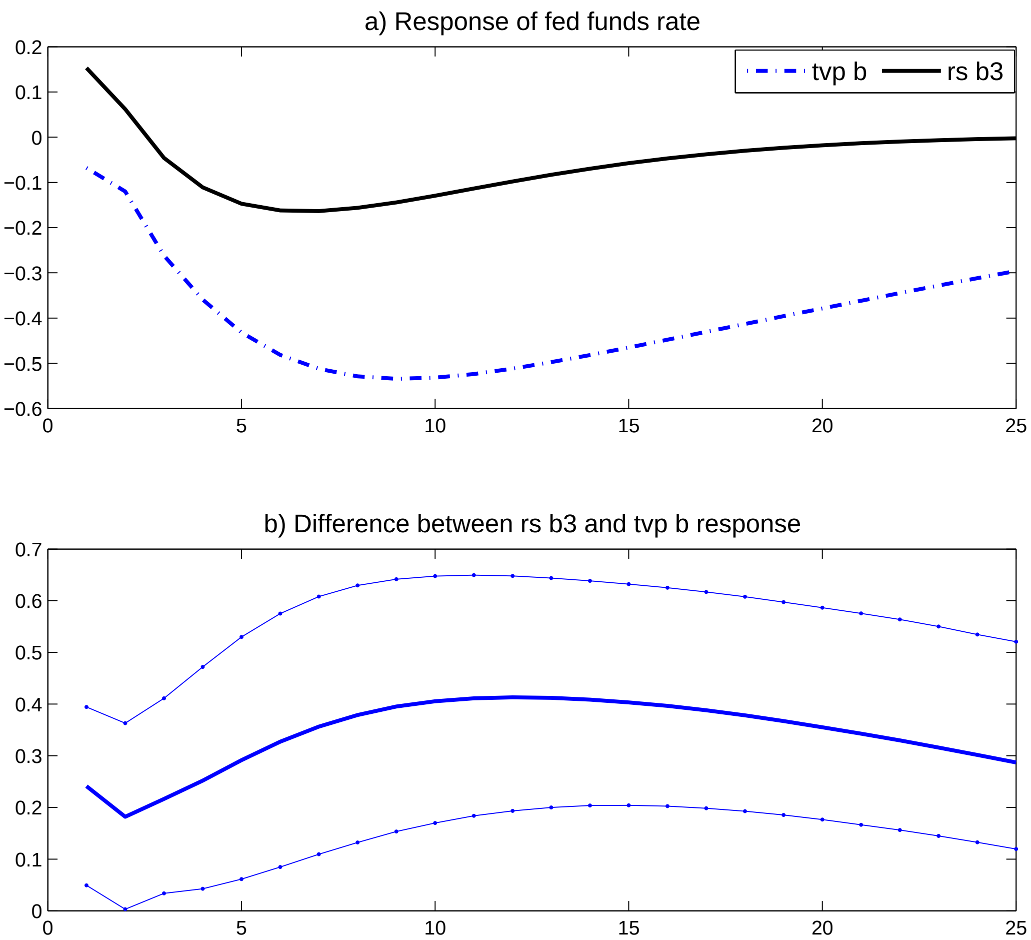
The figure shows the monetary policy shocks from the two best fit models. The dashed blue line is the tvp v model while the solid black line is the rs v3 model.

Figure 6: Long-run response to inflation



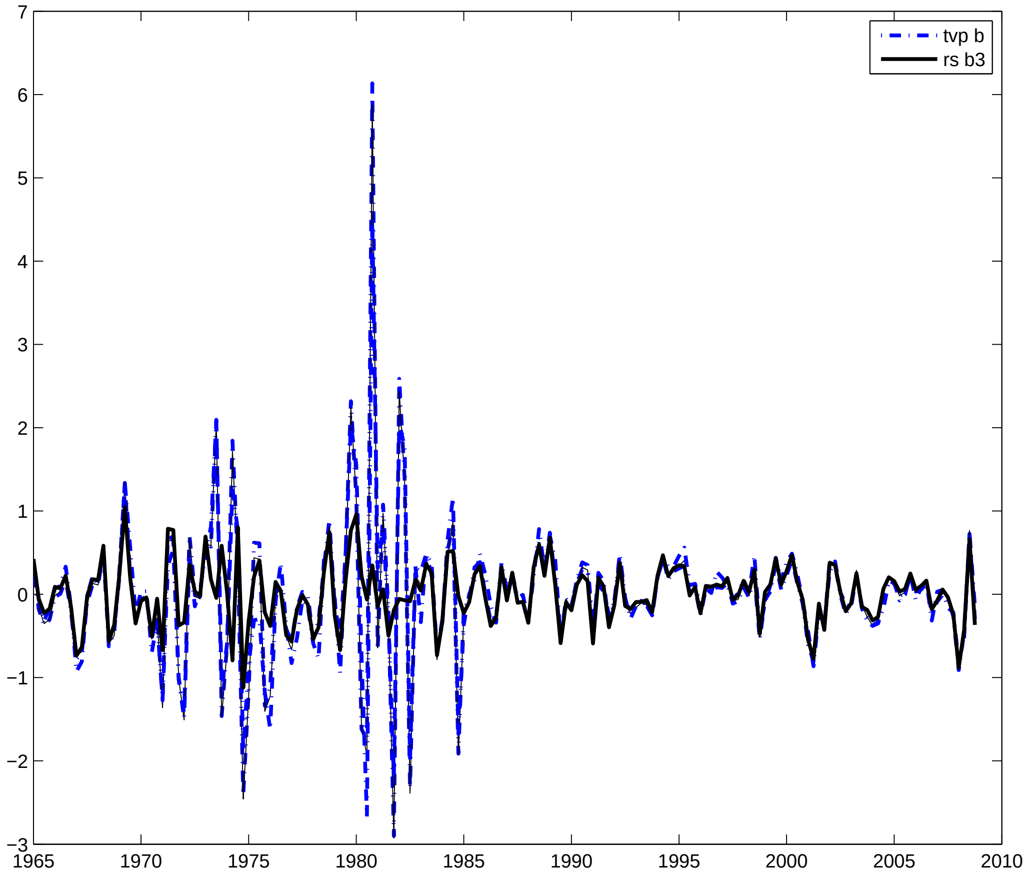
Panel (a): The green line shows the long run response to inflation in the tvp b model. The black, blue and red lines show the long-run responses to inflation for the three regimes in the rs b3 model. Panels (b),(c) and (d) show the corresponding posterior regime probabilities for the three regimes in the rs b3 model.

Figure 7: Inflation Shock: Response in 1982:Q3



Panel a) shows the impulse response of the fed funds rate to a one-unit inflation shock in 1982:Q3. The dashed blue line is the tvp b model while the solid black line is the rs b3 model. The solid line in panel (b) shows the difference between the two responses with the dotted lines showing 16th and 84th percentile credibility intervals for the difference.

Figure 8: Monetary Policy Shocks



The figure shows the monetary policy shocks from two models with change in policy coefficients. The dashed blue line is the tvp b model while the solid black line is the rs b3 model.